STUDENTS' USE OF INFORMAL REPRESENTATIONS IN PROOF CONSTRUCTION

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Mathematicians reason from informal representations (IRs), such as graphs, diagrams, and specific examples of more general concepts, when constructing proofs (Thurston 1994). Alcock (2004) found mathematicians to use *algebraic* IRs in their proof construction in order to understand statements, generate arguments, and check arguments. Samkoff et al. (2012) found mathematicians used *visual* IRs for noticing properties and generating conjectures, estimating the truth of an assertion, suggesting a proof approach, instantiating or representing an idea or assertion in a diagram, and validating theorems through diagrams.

As such, mathematics educators suggest that students should base proofs on informal reasoning (Garuti et al. 1998). Case studies, however, show instances of students both successfully (Sandefur et al. 2012) and unsuccessfully (Pedemonte 2007) constructing proofs based on IRs. This study addresses the following research questions: (1) To what extent do IRs help students write proofs? (2) When IRs are unhelpful in students' proof construction, why are they unhelpful? (3) Do students use IRs for the same purposes as mathematicians? (4) Are there common unproductive patterns of behavior in students' usage of IRs in proof construction tasks?

Twelve senior undergraduate mathematics majors participated in task-based interviews completing 7 calculus proof construction tasks. These 84 proof attempts were analyzed for the use of 221 IRs and the purposes for which those representations were employed. Overall, 21 (25%) of the 84 proof attempts were successful. Of the 70 proof attempts using IRs, only 10 (14%) were successful. Of the 14 proof attempts without IRs, 11 (79%) were successful.

Next, we used the frameworks by Alcock (2004) and Samkoff et al. (2012) to analyze the purposes for which students used IRs. Despite 11 out of 12 participants creating 48 algebraic IRs, only 6 IRs (13%) were used for purposes similar to the mathematicians in Alcock's (2004) study. In contrast, at least 7 participants used between 5% and 38% of the 173 visual IRs for the purposes described in Samkoff et al.'s (2012) study.

These data suggest not only that IRs were not necessarily helpful to students in proof construction tasks, but also that a minority of students attempts to use IRs for the same purposes as mathematicians. To investigate why these proof attempts were unsuccessful, we used an open coding scheme and found that students often use inappropriate IRs, make misleading generalizations from special cases, get trapped in time sinks while studying IRs, and use IRs to verify a truth of a statement rather than gain insight on why it is true. These results suggest that simply advising undergraduates to use IRs is not likely to be helpful in their proof construction.

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