SELF-EFFICACY BELIEFS OF HIGH SCHOOL STUDENTS IN AN ADVANCED COURSE IN PROOFS AND NUMBER THEORY

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In this study, we describe the effect of participating in an advanced course in proofs and number theory on gifted high school students' mathematics self-efficacy. The focus of the course was on learning how to prove theorems, and to that end, the instructor modeled higher order mathematical thinking with the intent of helping students develop mathematical habits of mind. We measured self-efficacy with pre- and post-surveys that included both Likert-scale items and open-ended question to determine ways self-efficacy was affected by course taking and whether effects influenced students' choice of college math courses.

Keywords: Affect, Emotion, Belief, and Attitudes, Reasoning and Proof, Advanced Mathematical Thinking

Introduction

The purpose of this study was to examine mathematical self-efficacy beliefs of high school students enrolled in an advanced mathematics course titled *Proofs and Problems in Number Theory and Algebra* (PPNTA). In this course, students were expected to engage in higher order mathematical thinking, develop skill in constructing proofs and showing counterexamples, participate in intellectual argument with their peers, and work both collaboratively and individually to solve difficult mathematical problems. Although the mathematically gifted students in this study generally have high math self-efficacy, their prior math experiences have emphasized calculation and content-specific objectives. This course was significantly different from the other advanced math courses students have completed because it focused on abstract properties such as elegance in proofs and metamathematics. We were particularly interested in studying self-efficacy in this context and with these students because of the way self-referent beliefs affect how students approach tasks, set goals, and persist when working through challenging tasks (Bandura, 1997).

According to Bandura (1994), self-efficacy refers to the beliefs individuals have about their ability to achieve a goal at a given level of performance. Self-efficacy affects the choices people make, and thus students with high self-efficacy typically make choices that lead to achievement, while students with low self-efficacy may engage in behaviors that undermine their success (Pajares, 1996).

Researchers have determined that gifted and high ability students typically have high self-efficacy and have a more accurate perception of their ability than do students of average aptitude. In our earlier studies (Hendricks & Millman, 2012), we found statistically significant differences in gifted students' pre and post course math self-efficacy. Further, there was less fluctuation in self-efficacy for students who were comfortable with the abstract nature of the course, whereas students who were more comfortable studying concrete concepts said changes in their self-efficacy were dependent on the difficulty of individual topics covered during the course. Though Pajares (1996) explains that gifted students' self-efficacy relies more on ability than performance, our findings did not bear that out. Rather, our results revealed that mathematically gifted students' comfort level with the abstract nature of the course was a better predictor of self-efficacy than was cognitive ability, which was high and consistent across students in the course.

Methods

PPNTA Course

The course was developed in partnership between Georgia Tech's Center for Education Integrating Science, Math, and Computing (CEISMC) and a public charter high school with an emphasis on math, science, and technology. Funded by Georgia's Race to the Top award from the U.S. Department of Education, the course was taught by a Georgia Tech doctoral student in Industrial and Systems Engineering studying Algorithms, Combinatorics, and Optimization. CEISMC's director (a co-author of this paper and professor of mathematics) collaborated with the course instructor to plan the PPNTA course and made several classroom observations during the semester and delivered lectures. The intent of the 18-week course was to help students discover that mathematics is not based solely on memorization nor is it fundamentally about calculation. To accomplish this, we created an *explore, generalize, prove, think* environment that parallels the culture of that of mathematics researcher. The course included topics such as equivalence relations, topology, and elementary Lie groups—mathematical ideas that go beyond the abstraction of modular arithmetic. Other course topics are basic properties of integers; divisibility and prime numbers; the Fundamental Theorem of Arithmetic; linear Diophantine equations; equivalence relations and their applications; basic properties of polynomials; divisibility of polynomials, divisibility methods, and roots of polynomials; combinatorics; elementary group theory; public key cryptography; and problem solving with computer programming. In studying these topics, students were expected to (a) identify what makes a mathematical proof correct, (b) learn commonly applied proof techniques, (c) develop proficiency in reading and writing mathematics in general and as proofs or counterexamples, and (d) apply problem-solving methods to find solutions and demonstrate the accuracy of their methods.

Participants and Setting

Twenty-eight 12^{th} grade students—18 males and 10 females—were enrolled in the course. Eleven of these students had parental consent and/or assented to be in this study. Of these students 9 completed both the pre- and post-surveys, and our results are based on these students. Six were male, and three were female. Four students were Caucasian (44.4%), 3 were Asian (33.3%), and 1 student each were Hispanic and Middle Eastern.

The study took place at a public charter high school that emphasizes math, science, and technology. Approximately 600 students are enrolled in grades 9 through 12, including 40% Asian, 29% Caucasian, 18% African-American, and 8% Hispanic. The majority of students (74.5%) are in gifted programs. The school offers 17 Advanced Placement (AP) courses, including those in calculus, statistics, physics, biology, chemistry, computer science, and humanities. Our participants were high ability students, and most had completed college-level calculus before enrolling in PPNTA. At the beginning of the course we asked students about their general mathematics capability; 4 said they were *extremely capable*, and 5 said they were *capable*. Five students intended to pursue a university degree in engineering, including 2 double majors in computer science/electrical engineering, 1 in biomedical engineering, one in mechanical engineering, and 1 in general engineering. Two students planned to pursue degrees in computer science, and 2 planned to study chemistry.

Data Collection

Changes in students' mathematical self-efficacy were measured using pre and post self-efficacy scales we developed for an earlier study (Hendricks & Millman, 2012). Items were developed based on Bandura's *Guide for Constructing Self-Efficacy Scales* (2006) and were aligned to course objectives. For example, students rated their confidence in areas such as *identifying fallacious* *reasoning in proofs* and *learning different methods to construct proofs*. Following Bandura's guide, pre- and post-survey items related to what students *can do* to measure their perceived capability. On pre-surveys, students also were asked to rank their confidence in achieving each course goal. To clarify, on the pre-survey, students ranked both their current ability for each course goal as well as their confidence that they would be able to reach each goal.

The pretest, given near the beginning of the course, included 13 items related to course goals. For each item, students ranked their self-efficacy on a scale from 0 (low self-efficacy) to 100 (high self-efficacy) for both their current ability for the course goal (*How confident are you that you can already construct valid proofs?)* as well as their confidence they would be able to reach the goal (*How confident are you that you can learn to construct valid proofs?*). The post-survey contained identical items but students only ranked their current ability. The post-survey also contained 11 items that related to students' experience in the course.

Results and Discussion

We compared students' self-efficacy beliefs at the beginning of the course (pre self-efficacy), their self-efficacy related to reaching course goals (potential capability), and their self-efficacy at the end of the course (post self-efficacy). Because of the small sample size (n=9), we have provided descriptive statistics only. Students' average pre self-efficacy (SE) across all SE items was 75.5, and their average potential capability was 88.7. However, on the post-survey, average SE was 78.0, which was almost 11 points lower than their estimated potential and just 2.5 points higher than their pre-SE. We saw positive pre-post changes in SE on 8 items, negative changes on 4 items, and no change on 1 item (see Table 1). SE increased most on the items *Create examples that provide insight into designing proofs* (+11) *Learn different methods to construct proofs*. (+9.7), *Construct valid proofs* (+8.8) and *Develop a mathematical habit of the mind* (+8.1). The greatest areas of decrease were on items *Engage in intellectual arguments with others about mathematics* (-6.3) and *Work in teams to solve mathematical problems from number theory and algebra* (-5.9).

Seven of the 9 participants said their mathematics confidence had increased due to their experiences in the course. One student's confidence decreased and another's stayed the same. Students whose confidence increased said they learned to think critically and reason through problems. One student explained, "*I felt like every problem we encountered in this class was a familiar one, but we gained a new insight on how that problem is solved or why a theorem works*." Another wrote, " *I had the chance to examine the reasoning behind the math that I've been using and understand why the formulas and theories work*."

In describing the course a student stated, "I *learned much more about how mathematics works than in any other class. I enjoyed all of the problem sets because they felt more like a challenge rather than another 'problem' to solve*." Other students also described the challenging nature of the course. One student, for example, stated "*It was a bit difficult to follow sometimes because there was a lot to think about all at once but it was cool once I got it*." Another said the course "…*was very frustrating when I fell behind and did not know what was going on, but those moments when I finally [understood] a problem were interesting and fulfilling, maybe even fun*."

Three students (two females and one male) said taking the course sparked their interest in taking similar courses in college. A female student wrote, "*Even though I struggled, I know this kind of math exists and it can help me decide what kind of class I want to take in college*." A male student planning on a double major in chemistry and math wrote, "*The course sparked my interest in proofs and other fields of rigorous mathematics, and I am excited to take more advanced math courses in college*."

Table 1. SE Changes Pre, Capability, & Post

	Pre	Capability	Post
Learn different methods to construct proofs.	70.0	87.1	79.7
Understand the concept of "elegance" in proofs.	69.4	84.2	73.1
Create examples that provide insight into designing proofs.	61.7	80.9	72.7
Construct valid proofs.	69.4	84.4	78.2
Identify the fallacious reasoning in incorrect proofs.	76.9	91.9	79.1
Use concepts learned about elementary number theory and algebra in other courses to solve problems in THIS course.	78.0	92.4	78.4
Define what a "mathematical habit of the mind" means.	74.8	89.2	79.8
Develop a mathematical habit of the mind.	68.1	78.7	76.2
Engage in intellectual arguments with others about mathematics.	88.1	96.8	81.8
Explain ideas that motivate your proofs.	79.1	93.0	77.2
Work individually to solve mathematical problems from number theory and algebra.	79.2	87.2	77.0
Work in teams to solve mathematical problems from number theory and algebra.	84.1	93.1	78.2

As described by other students in previous semesters, students both enjoyed and were challenged by this course. For example, when asked what she liked best and least about the course, a student said, "*Understanding a problem after working on it for a long time; NOT understanding a problem after working on it for a long time*." Another student wrote, "*I liked it best when I was completing a proof and I'd look back on it and marvel at how elegant and simple it all was in hindsight. I liked it least when I found myself completely stumped by a difficult proof.*"

Though our sample size was small, results of this study are similar to our findings with two other groups of students. Students found the course challenging, they tended to overestimate their capability, but completing the course increased most students' mathematical self-efficacy. Further, a third of the participants in this study, including two females, said their experience in the course had affected their decisions about the math courses they will take in college. As enrollment for the course continues to increase, our goal is to replicate the study with a larger group of students. Further, we plan to track students into college to determine whether they do indeed choose to take advanced math courses based on their experiences in the PPNTA course.

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References

Hendricks, C., & Millman, R.S. (2012). *Effects of an advanced high school course in number theory on students' mathematical self-efficacy*. Paper presented at the annual Psychology of Mathematics Education-North America Conference, Reno, NV.

Bandura, A. (1994). Self-efficacy. In V. S. Ramachaudran (Ed.), *Encyclopedia of human behavior* (Vol. 4, pp. 71-81). New York: Academic Press.

Bandura, A. (1997). *Self-efficacy: The exercise of control.* New York: Freeman.

Bandura, A. (2006). Guide for constructing self-efficacy scales. In T. Urdan & F. Pajares (Eds.), *Self-efficacy beliefs of adolescents* (pp. 207-337). Greenwich, CT: Information Age Publishing.

Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students. *Contemporary Educational Psychology, 21,* 325-344.