INSTRUMENTED ACTIVITY AND SEMIOTIC MEDIATION: TWO FRAMES TO DESCRIBE THE CONJECTURE CONSTRUCTION PROCESS AS CURRICULAR ORGANIZER

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We document part of the process through which conjectures produced by students, with the aid of the dynamic geometry software Cabri, when they solve proposed geometric problems, become a curriculum organizer in the classroom. We first focus on characterizing students' instrumented activity recurring to utilization schema (Rabardel, 1995; in Bartolini Bussi and Mariotti, 2008), and then describe the teacher's content management through which the ideas produced by the students become key elements of knowledge construction.

INTRODUCTION

During the years 2010 and 2011, we carried out a formal study[1] of pre-service mathematics teachers' conjecture production and the content organization of their second semester plane geometry course (Universidad Pedagógica Nacional, Colombia) based on these conjectures. This academic effort continues our research interest which, since 2004, consists of delving into issues related to teaching and learning proof in geometry at tertiary level. In this paper we want to present the tool we use to analyze how the formulated conjectures become an organizer of the implemented curriculum, favoring student participation in its construction. Initially, we present our framework that includes not only two theoretic references on which our analysis is based, instrumental approximation and teacher semiotic mediation, but also what we mean by "curricular organizer". We then present the research methodology, and we briefly describe the experimental device. Following, we give a succinct description of the categories that we constructed to analyze students' instrumented activity and the teacher's mediation activity. We then present an example in which the categories were used to analyze the activity concerning a particular problem. Finally, we discuss the achievements and projections of our study.

THEORETIC FRAMEWORK

It is our intention to try to articulate, imitating what Arzarello and Paola (2008) do, two theoretic references that are useful to deepen into issues that concern teaching and learning proof: instrumental approximation that becomes a lens to provide information about the use of dynamic geometry as an effective instrument for student conjecture production; and the teacher's semiotic mediation that gives insight to the teacher's role

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when the intention is to make students' productions, personal meanings, evolve towards mathematical meanings them to obtain the statement they propose.

Instrumented activity

We follow the widely accepted premise that learning is mediated by artifacts. Interpreting Rabardel (1995, in Bartolini Bussi and Mariotti, 2008), we consider an *artifact* to be any material or symbolic object created by man with a specific aim.; From our point of view, artifacts, as a product of human activity, are perceptible, can be analyzed, and have physical existence independent of the situation from which they originate, be it as an object itself or through a register.

So that an artifact can intervene in knowledge generation it must become an instrument for the person learning. In Rabardel's theory, the notion of *instrument* includes an artifact (or part of an artifact or some artifacts) together with utilization schemes, that is, an observer's interpretation of actions "relative to the management of the characteristics and particular properties of the artifact" or that are "means for the achievement" of a task (Rabardel, 2011/1995, p. 171). According to the researcher, making an artifact an instrument requires the articulation of two processes, in what he calls *instrumental genesis.* These are: (i) *instrumentalization* that is, progressive acknowledgement of the possibilities and limitations of the artifact and its different components; and (ii) *instrumentation*, or appearance and development of utilization schemes. Our interest in the theory of instrumented activity lies in that the utilization schemes can be seen as signals of mathematical activity carried out by the students when they solve problems, aided by the use of Cabri, and propose conjectures as a result of their work. They provide us information about the personal and contextualized experiences with which the students give meaning to the problem's statement and the objects involved in the situation, propose a strategy to solve the problem, and formulate conjectures as the solution. As the artifact is used varied signs are produced the capture (encapsulate) the actions of the instrumented activity and the ideas that sprout during these or associated to these. These signs evidence personal meanings with which the teacher can carry out the semiotic mediation of the geometric content that students should understand and learn.

Teacher's semiotic mediation

In our study, the analysis of the teacher's semiotic mediation is focused on the treatment of the signs derived from the use of the artifact Cabri when solving problems for which students have to produce a conjecture. These signs include geometric Cabri figures and statements with mathematical content, related to the conjecture, produced by them. The signs are considered as entities that represent something for someone; they reflect internal cognitive processes and are mental tools for doing particular tasks.

According to Mariotti (2012), the principal characteristic of the signs derived from the use of an artifact, in the mathematics education field, is its strong link with the actions done with it. We recognize that the artifacts have *semiotic potential* (Bartolini Bussi and Mariotti, 2008) that is evidenced in so far the *utilization schemes* and the *signs* are used intentionally for mediation. The teacher's semiotic mediation is based on the profit he/she can get from the signs produced by the students to propitiate, foment and affect the relation between the students and mathematical knowledge.

Therefore, the teacher's responsibility is to design strategies that connect the individual and social perspectives, and to act on the cognitive and metacognitive levels. The semiotic mediation related to the production of conjectures in proving activity can be evidenced in actions centered on: I. The conceptualization of objects and relations; II. The understanding and use of conditional statements; III. The conjecture as solution to the problem; IV. The theorem looked for; V. The conformation of the notion of theorem, postulate, definition. In all of these, the teacher's semiotic mediation process relies on the experience lived by the students, on his/her own observations and formulations, to give meaning to the mathematics statements that emerge.

Problems-conjectures-theoretic system as curricular organizer curricular

Rico (1997, p. 45) introduces the notion of curricular organizer to refer to "the knowledge we adopt as fundamental component to articulate the design, development and evaluation of didactic units". For the author, a necessary condition for accepting a type of knowledge as a curriculum organizer must be its objective character and the diversity of options it generates. It must offer a conceptual framework for teaching mathematics and a space for reflection that shows the complexity of the transmission and construction of mathematical knowledge processes, and criteria to approach and control that complexity. In our methodological approach for teaching proof in the tertiary level, we have used a composition of three elements as curriculum organizer which we denote by *problems-conjectures-theoretic system*. To characterize it, we begin by detailing its elements and the relations among them.

The problems are of a geometric nature; to solve them empiric exploration is required for which the use of dynamic geometry is permitted, and a conjecture must be formulated; one or more possible theorems of the theoretic system underlie each problem. The problems proposed to the students are of two types: of *suggested construction*[2] and of *creative construction*[3]. Student's conjectures are the result of exploring the situation given in the problem, using dynamic geometry software, and have a high probability of being acceptable proposals from a theoretic point of view. The theoretic system is the framework for the problem's solution; the problem can be represented and tackled with the available geometric content. The conjectures that originate from solving the problem extend the theoretic system if they become theorems. In our case, the guideline of the theoretic system is Birkhoff's model (1932) for Euclidian geometry, in which "the facts embodied in the scale and protractor" are introduced (p. 329).

[2] The representation of the situation described is based exclusively on the construction of the objects that satisfy the conditions given in the problem itself and the search for invariants is based on the direct exploration of the objects represented (constructed).

[3] They require auxiliary constructions that provide the necessary geometric conditions to determine, via explorations, the existence of an object (generic or specific). Therefore, the representation of the situation is based not only on the construction of objects that satisfy the given conditions but also those that permit solving the problem.

Given the above description it is relatively easy to see that in the composite *problems-conjectures-theoretic system* the geometric content occupies an outstanding position; it is natural to imagine that if it is used as a prescriptive curriculum organizer it will not present any novelty with respect to the traditional way of presenting and/or studying mathematical content. But, as implemented curriculum, what is actually done as response or reaction to the situated events and a specific time and place, our proposal is different from other organizers because the students' voices are the key element during the collective study of their conjectures.

RESEARCH METHDOLOGY

Our research study is framed in the teaching experiment methodology (Cobb, 2000): events of experimental teaching are analyzed using a certain theoretical reference with the purpose of identifying phenomena of interest. The research design has as starting point the transcriptions of some Plane Geometry classroom sessions, course developed during the first semester of 2007.

To study the instrumented activity with Cabri, we identified, in the students' interventions, issues that became analysis categories (instrumentalization, instrumentation-utilization schemes, artifact semiotic potential, geometric figure-signs, and statement-signs). The knowledge we had of the program and, based on frequent observation of students' performance when using Cabri to solve problems, we proposed, a priori, a set of utilization schemes with respect to constructions and the use of the dragging function; other schemes arose as we carried out our analysis. With the objective of analyzing the teacher's semiotic mediation in the conjecture treatment in class, and this way document the process through which the ideas produced by the students become key elements of knowledge construction, the teacher's actions were coded and grouped into the five categories previously mentioned.

Analysis examples

To illustrate how we did the analysis, we present two examples, one relative to the students' activity and the other to the teacher's mediation.

Students' activity

We present the analysis of part of a pair of student's (Nan and Ing) activity when solving a *creative construction problem*[4]. The theorem we expected to obtain with this problem is the existence of a perpendicular line to a given line through a point of that line. It results as a theoretic necessity to be able to construct the complementary angle and justify its existence. Even though an exploration based solely on measuring and dragging is useful to notice that point E exists, we expected students to geometrically characterize the point in their conjecture. That requires an auxiliary construction that provides the sufficient geometric conditions for such an existence.

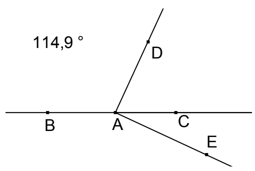
[4] The problem statement is: let $\overrightarrow{AB}$ and $\overrightarrow{AC}$ be opposite rays and $\overrightarrow{AD}$ another ray. Does there exist a point E in a half-plane determined by $\overrightarrow{AB}$ in which D is found such that ∠CAE and ∠DAB are complementary?

The group's activity can be divided into six phases: (i) construction of opposite rays, (ii) construction of $\overrightarrow{AD}$, (iii) dragging $\overrightarrow{AD}$ to explore, (iv) construction of $\angle EAC$, (v) dragging $\overrightarrow{AD}$ to verify the construction, (vi) writing the conjecture. In these phases, the following construction and dragging utilization schemes were inferred:

Scheme	Code	Phase	Description
Opposite rays construction	C7	i	They construct a line, use the Ray option and construct opposite rays with the same origin point on the line.
	C11		They construct two rays in opposite directions that perceptually seem to be colinear.
Ray construction	C5	ii	They use the Ray option, click in two different places on the screen and determine a point on the ray.
Construction of complementary angle	C18	iv	They measure the initial angle, calculate the difference between that measurement and 90, construct a ray and rotate it around the endpoint according to the calculated difference.
AETO		iii	They drag a point or object to sweep out and explore a more or less complete region.
AVCXMO		v	They drag a point or object to place them in extreme situations and verify the construction
AVCEMO			They drag a point or object to place it in special situatiions and verify the construction.

Table 1: Construction and dragging schemes: Nan and Ign

The following transcription corresponds to phase v. Once the students constructed $\angle CAE$ (C18), they use dragging to verify that this angle is always the complement of $\angle BAD$. The students' dialogue during this phase is the following one:



1 Ign: Ready. And it satisfies the dragging test. Yes. Let's drag. Okay?

2 Nan: Drag D.

3 Ign: D? Ah! Yes. Because E is the dependent one. [Drags $\overrightarrow{AD}$ bringing it close to $\overrightarrow{AB}$.]

4 Nan: Make it more tan ninety [referring to the measure of $\angle BAD$.]

5 Ign: [Rotates $\overrightarrow{AD}$ in the negative direction in such a way that E ends up in the other half-plane.]

Initially, Ign's dragging corresponds to the utilization scheme AVCXMO, because he moves $\overrightarrow{AD}$ to positions close to $\overrightarrow{AB}$ [3]. Later, to satisfy Nan's request, he drags the ray until it is in a position for which $m\angle BAD$ is more than ninety, that is, he considers a special position and therefore uses scheme AVCEMO [5]. They do not realize that when they drag, point E is in a different half-plane than where D is fact that does not correspond to the problem's statement. They have not yet understood the conditions stated in the problem because they consider angles DAB that are not acute. The utilization scheme used has semiotic potential to inquire about their concept of complementary angles. In this fragment, we observe that the instrumentalization stage in which Nan and Ign are permits them to easily recognize the independent and dependent points in their construction. Moreover, Ign's intervention "And it satisfies the dragging test" [1] is another evidence of his level of instrumentalization. The conjecture they finally express (mathematical sign) is: *If you have $\overrightarrow{AB}$ and $\overrightarrow{AC}$ opposite and $\overrightarrow{AD}$ another ray such that the measure of $\angle BAD$ is acute, and $\angle CAE$, with E in the same half-plane in which D is, so that they are complementary, then $\angle EAD$ is right.*

Teacher's mediation

To analyze the teacher's presentation of the conjectures produced by the students and the respective commentaries, we fragmented her intervention in moments. For each fragment we specified the intention that we could discern or that she makes explicit, and also the actions through which she manages her intentions. She finds in each conjecture some element worthy of an analysis that will contribute something to understand the situation itself, some associated theoretic element, or mathematical issues. We present part of the analysis of Fragment 14 as an example of the use of the codes –teacher's actions. These will be written in *italic* and in parenthesis the Roman numeral that indicates the general category in which the action is found.

The teacher analyzes the conjecture of one group. She points out that they propose a conjecture that is different from those analyzed in previous fragments[5], because their conclusion is not that complementary angles but that $\overrightarrow{AE}$ is perpendicular to $\overrightarrow{AD}$. To analyze the conjecture she compares it with the one obtained as a consequence of the analysis of the previous conjectures[6], from which the expected theorem arises something she *emphasizes as a contribution to the theoretical system* (IV). The teacher indicates she has *identified that the conjecture reported is in agreement with what was constructed and with the conclusion* (II) and at the same *realized theoretic control* (III) in so far as she mentions that the student's report of the conditions they constructed is valid. She *emphasizes that there is an imprecision* (IV) in the conjecture and a member of the group immediately recognizes the data that is missing in the hypothesis: not mentioning that point E is in the same half-plane, determined by $\overleftrightarrow{AB}$, in which D is. We conclude that student's sign evolved maybe due to the teacher's mediation process carried out with the previous conjectures.

[5] Given two complementary angles, coplanar, that share the same vertex and one of its sides is the opposite ray of one side of the other angle then the angle formed by the other two rays is right.

[6] The conjecture is: If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays and $\overrightarrow{AD}$ is another ray, and let $\overrightarrow{AE}$ such that $m\angle EAD = 90$ then $\angle DAB$ is complementary to $\angle EAC$.

The teacher starts an analysis by comparing the conditions exposed in the hypothesis and thesis of both conjectures, as resource to aid in the comprehension of the difference between them. As she indicates that the thesis and one of the conditions included in the hypothesis are interchanged, she *emphasizes the fundamental elements of a conditional statement* (II). She then classifies Nan and Ign's conjecture, where the conditions are complete, as a theorem indicating with this that she aproves the statement because it reports the dependency evidenced in Cabri and a result that can be proved within the theoretic system they have on hand. The teacher emphasizes that this conjecture *indicates the constructed properties and those discovered* (II).

FINAL COMENTARIES

The detailed study of the transcriptions of the students' work was a useful because it permitted us to describe and analyze their actions to solve a problem and formulate conjectures; infer and categorize utilization schemes related to the artifact Cabri that students bring into play, according to the type of problem; identify different types of signs produced by them; amplify Mariotti's proposal (2012) to determine the semiotic potential of the artifact via the schemes; and establish a correlation between the schemes and the personal signs produced by students when solving a problem. Also, since the teacher was committed to the curriculum organizer *problems-conjectures-theoretic system*, the transcriptions of the socialization, guided by the teacher, where the conjectures are discussed, permitted us to identify the teacher's semiotic mediation actions directed towards producing a mathematical sign (theorem statement) that corresponds to the one that she wants to include in the theoretic system once it is proven. Besides, we were able to establish a categorization and a frequency count of the teacher's semiotic mediation actions referred to the central object of mediation: conceptualization, conditional statements, problem solution, theorems and metamathematic notions. With the proposed semiotic mediation categories, we contribute methodologically to the corresponding study when the teacher centers on mathematical content derived from the problem solving process. This categorization differs from the one proposed by Mariotti (2012), since hers is based on actions realized with the artifact. The semiotic potential of the artifact, via the utilization schemes identified, is very close to the mathematical.

Also, the classification of the utilization schemes permitted us to define and group them into two categories: construction schemes and dragging schemes for exploration or verification. With our analysis we could establish links between the theory of semiotic mediation and instrumental genesis, but it was not possible to use them to extend the connection, due to time limitations. Our next step is to carry out another experimental design to determine if the products we have so far are useful to propose a model that ensures getting the most out of the semiotic potential of Cabri and ensuring the legitimate participation of the students in constructing knowledge during the teacher's semiotic mediation process.

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