A MODEL TO INTERPRET ELEMENTARY-SCHOOL STUDENTS' MATHEMATICAL ARGUMENTS

Chepina Rumsey

Kansas State University

The goal of this study was to develop and use a modified coding system and model of argumentation to investigate and characterize fourth-grade students' mathematical arguments during an eight-lesson, whole-class teaching experiment regarding the arithmetic number properties. Sixty-seven instances of student arguments were identified using a model adapted from Toulmin's (1958) model of argumentation and Stylianides' (2007) elements of argumentation. In this paper, I document the development of the modified coding scheme and model, present the relevant elements of the model of argumentation, and briefly interpret the characteristics of students' early arguments, which will be further discussed in the presentation.

PURPOSE OF THE STUDY

Many students are progressing through school with the false impression that mathematics is answer-driven and static rather than a dynamic subject area open for discovery. Computing seems to be an instructional priority despite the fact that the field of mathematics goes far beyond procedures. Mathematics involves exploring, conjecturing, and justifying, which are also important components of other subjects (e.g., science, language arts, and history). There have been calls for conceptual understanding (Kilpatrick, Swafford, & Findell, 2001) and mathematical argumentation in recent American mathematics education documents, which require going beyond computation at the elementary level.

It is well recognized by researchers that at the elementary level, students are capable of going beyond computation in order to justify mathematical concepts (e.g., Stylianides, 2007; Keith, 2006; Carpenter, Franke, & Levi, 2003; Ball, Lewis, Thames, 2008). Although justifying and arguing are seen by many researchers in the field to be vital aspects of elementary school curriculum, "little research has focused on the issue of understanding and characterizing the notion of proof at the elementary level" (Stylianides, 2007, p. 1). We need information about early reasoning and proof if we are to help students transition along the continuum from early mathematics and informal proof toward grade levels requiring formal proof. If educators aim to include more argumentation in the elementary classroom in order to teach for conceptual understanding, they need to know what characterizes children's mathematical arguments. In this study, I taught lessons about the arithmetic properties to a classroom of fourth-grade students with instruction that promoted mathematical argumentation in order to develop a model highlighting the characteristics of students' early mathematical arguments for number properties. In this paper, I document the development of the modified coding scheme and model, present the relevant elements

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of the model of argumentation, and briefly describe the characteristics of students' arguments.

BACKGROUND

Students' use of empirical evidence is common and even at the college level, students are willing to accept examples as proof (Harel & Sowder, 1998). Consistent with research on empirical evidence, Keith (2006) noticed all of her second-grade students using examples to justify in November before shifting to a "generalized argument" (p. 64) later in the school year. Keith used Carpenter, Franke, and Levi's (2003) levels of justification (appeal to authority, justification by example, and generalizable arguments) to categorize students' justifications and showed that over the school year there was a shift away from examples toward generalizable arguments. In Keith's (2006) study the students wrote conjectures, but the analysis is mostly limited to the justifications since that was the focus of her research study. In studies regarding elementary school students' justifications, the claim is often an implicit part of the argument even though "in mathematics, the making of claims is central" (Ball, Lewis, & Thames, 2008). Ball, et al. investigated the process of making conjectures as part of three essential practices: "naming and using names, making and interpreting claims, and evaluating mathematical assertions" (p. 41). In another study, Stylianides (2007) analyzed third-grade students' arguments about even and odd numbers in the context of four elements of argumentation (foundation, formulation, representation, and social dimension) in order to determine if the arguments could qualify as proofs. In his study, and similarly with other studies, the students explored a problem chosen by the teacher and investigated conjectures together as a class and thus, did not need to state the claims explicitly since everyone was part of the same conversation. My research extends the importance of claims in mathematical argumentation while investigating and characterizing the other vital components of a mathematical argument at the upper-elementary school level.

THEORETICAL PERSPECTIVES

I view mathematics learning as a construction of ideas through social interactions wherein novices are brought in to a larger mathematics culture and community. Consistent with this view, the *emergent perspective* (Cobb, 2000; Yackel & Cobb, 1996) provided an overarching framework for this research, in which I taught lessons and analyzed the students' arguments. Following the emergent perspective, learning can be examined along two dimensions: the social and the psychological. The social aspects informed my development of the lessons and my understanding of the students' learning through the sociomathematical norms and classroom mathematical practices that developed during a whole-class teaching experiment. I examined the psychological aspects through the lens of argumentation that was informed by Toulmin's (1958) model of argumentation and Stylianides' (2007) four elements of argumentation, which I will describe below.

Toulmin's Model of Argumentation.

Toulmin (1958) attempted to identify the universal and field-dependent elements of an argument and describe the layout of all well-constructed arguments. He wrote that all well-constructed, rational arguments consist of three main interrelated elements: the claim, data, and warrant. The claim is the conclusion being justified and the data consist of the facts that build the foundation for the claim. The warrant is the element that builds a bridge from the data to the claim, which shows that the conclusion is valid. He proposed three auxiliary components (modal qualifier, backing, and rebuttal) that may be present in arguments, although they are not essential elements. Toulmin's model can be used to both construct a claim and examine the claims of others and it has been used across disciplines in order to identify, create, and evaluate arguments in science education, mathematics education (e.g., Pedemonte, 2007), and language arts. Specifically for mathematics education, Pedemonte found that using Toulmin's model was an essential tool for comparing the structure between students' arguments and mathematical proofs.

Stylianides' Four Elements of Argumentation.

Stylianides (2007) was interested in the characteristics that determine if an argument can count as a proof at the elementary level and he defined the four elements of an argument as the following: (a) foundation, (b) formulation, (c) representation, and (d) social dimension. Stylianides developed his model based on his observations of the development of formal mathematical proofs. According to Stylianides, proofs are developed using definitions to form a solid foundation. Next, logical arguments are formed and arguments are represented using a mathematical language. Finally, a proof being accepted as such relies on the opinions of the mathematical community.

METHODOLOGY

A basic qualitative research methodology was appropriate for this study, where the focus was on collecting, analyzing, and characterizing students' arguments about number properties during a sequence of instruction that promoted mathematical argumentation. Specifically, the study employed a whole-class teaching experiment (TE) methodology set in the fourth-grade mathematics curriculum, which was focused on some of the arithmetic properties, in order to promote instruction that included mathematical argumentation. The goal was to model and characterize the students' mathematical arguments. Without capturing and describing what the individuals constructed with regard to mathematical argumentation after interacting with their peers and teachers, "there would be no basis for coming to understand the powerful mathematical concepts and operations students construct" (Steffe & Thompson, 2000, p. 267).

The participants of the study included a fourth-grade classroom with 22 students at a small, suburban elementary school in the midwestern United States. There were 463 students enrolled in the elementary school. The majority of the students at the school were white/non-Hispanic (96.1%) and about a quarter of the students (24%) received free and reduced lunch. I volunteered in the classroom before the teaching experiment

began and the students were thus familiar and comfortable with me as their teacher for the TE lessons.

Teaching Experiment Lessons.

I taught the eight, non-consecutive TE lessons over a period of five weeks and utilized the existing textbook and lesson plans with modifications drawn from the literature on promoting argumentation and teaching number properties. In the textbook, fourth-grade students are introduced to properties of multiplication (commutative property, identity property, zero property, and the associative property) as vocabulary words and diagrams, are given examples to examine, and then assigned exercises to practice the specific property addressed in each section. The properties in the textbook chapter are taught along with multiplication facts in order to show efficient ways of multiplying (e.g., that changing order does not matter and to generate fact families), and are not taught in their own right as key elements of mathematics.

The TE lessons were related to the classroom textbook, but included more opportunities for argumentation and justification. Modifications to the textbook lessons included (a) supplementary problems, number sentences, and counter-examples and (b) an emphasis on explaining and exploring the arithmetic properties using the language of argumentation. Promoting a discussion about number properties was an important component of the modifications and true/false and open number sentences provided opportunities for discussing numerical equality.

Data Collection and Analysis.

In order to analyze and characterize the students' arguments, I audio- and video-recorded each of the TE lessons, transcribed them, and highlighted instances of arguments. I eliminated sections of the lesson transcripts that related to organizing and gathering the class to the front of the room, daily classroom dialog unrelated to mathematical arguments, behavior issues, and computational tasks that did not require justification. Keeping the models from Toulmin (1958) and Stylianides (2007) in mind, I developed a modified coding model that better described student argumentation at the Grade 4 level while still maintaining the overall structure of their analytical tools. I discuss this modified model and coding scheme in detail in the next section.

Using the coding structure, I identified 67 student arguments and noted what was happening in each clip and the recurring themes. Characterizing what was occurring in the students' mathematical statements involved looking at the statements in context rather than looking at lines of transcript unconnected to what was happening in the classroom.

Each argument was coded using the key words and definitions related to the elements of argumentation that I identified empirically when open coding for themes in the highlighted arguments. After modifying the definitions and adding an example for each keyword, I randomly selected eight arguments to check with another coder in order to see how another person interpreted the keywords and definitions. The other

coder and I compared codes, I revised the definitions of the keywords, and then I randomly selected 20 arguments to code and compare again. In the final round of coding and comparing, I randomly selected 20 arguments from the arguments that had not been coded. After all the inter-rater reliability coding, two people analyzed 48 (or about 72%) of the arguments. All of the clips were interpreted using the model developed through the coding and revisions, and then the codes were transferred to a spreadsheet so that the collection of 67 arguments could be sorted and analyzed.

Revised Coding Scheme.

When using Toulmin's model of argumentation, Klumpp (2006) wrote that some researchers have difficulty because it is common for them to neglect the role and context of statements. My goal was to investigate all parts of fourth-grade students' arguments and the interactions between them while also capturing the social dimension of the arguments. The social interaction is an essential part of an argument and I wanted this to be prominently represented in the model and analysis. Toulmin's (1958) closed model had an element in which challenges could be predicted, but there is not a place for actual challenges to be placed within the model to record the conversation that unfolded between students. Stylianides (2007) included a social element but the conversation and challenges did not play a prominent role. The students in my study did challenge each other and so I replaced Toulmin's *exceptions* with challenges to reflect what the students were doing.

As I looked at the open coding themes, I saw similarities between the arguments emerge that became the essential components of the final model used for coding and analysis. Examples, used for two purposes, were frequently used and so needed to have a place in the model. The term *background information,* rather than what

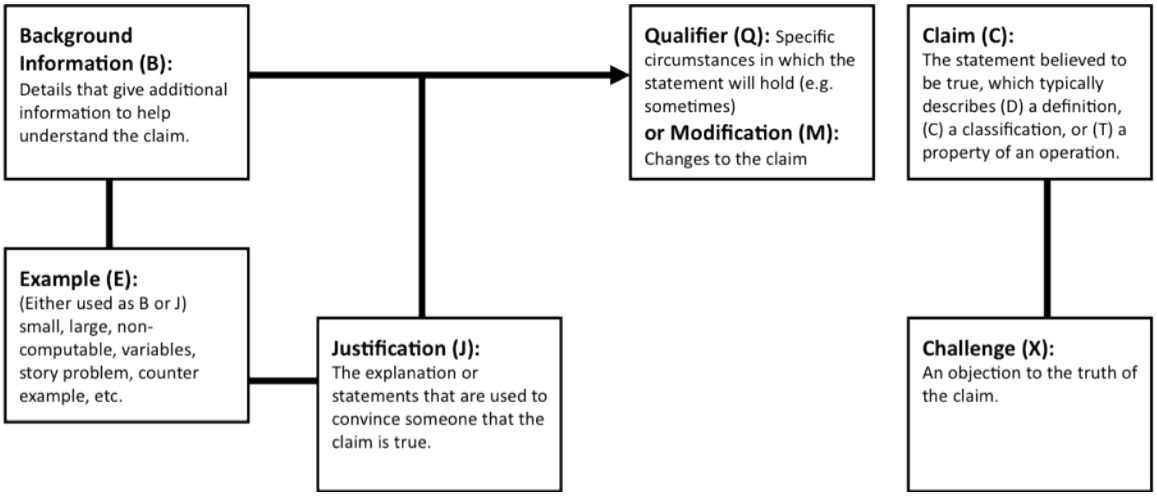

Figure 1: The modified model of argumentation

Toulmin called data, captured the students' language and thinking about this element in the study. The students used it to elaborate and clarify the claims. I also saw that qualifiers were relevant to this age group and went beyond the phrase "I think". In addition to qualifying their claims and the claims of others with phrases like "sometimes" or "when *a* and *b* are the equal", the students also modified the claims.

The six elements captured in the model (See Figure 1), used for coding and analysis, represent the students' arguments from my research and reflect current research about students' mathematical arguments.

GENERAL RESULTS RELATED TO ARGUMENTATION

In addition to the development of the coding scheme and model, there were other findings related to students' mathematical argumentation. From the data, I found that the students included many of the six elements within a single argument (claim, justification, qualifier, modification, challenge, background information, and examples). Even though this model has six main elements, only two out of the 67 student arguments contained all six in a single argument, yet every argument contained a claim. Both of these arguments occurred in the sixth lesson, when there was more small group-work time. This could suggest that either the small-group work promoted more elements to surface or that it took some time for the students to consider all of the elements in an argument. The data from this study provides a snapshot of students' early attempts at argumentation when it is emphasized in classroom lessons and so it is possible that students do not consider all of the elements until they have been exposed to argumentation for a longer period of time.

Specifically related to *claims*, out of the 83 claims in this research study, 17 were "classifications or definitions" and 66 were "properties of operations". About 95% of the claims that I provided as the teacher were properties of operations, yet about 63% of the students' claims were properties of operations and about 37% were classifications or definitions. As the teacher, I emphasized properties of operations, but the students found classifications and definitions valid claims for which to provide an argument. The three most common types of *justifications* used by the students were examples with small numbers (30% of all justifications), justifications with reasoning (about 28%), and examples with large numbers (about 12%). Only eight students out of 22 in the classroom provided *background information* as part of their mathematical argument. About 69% of the time, a student used background information to clarify his or her own claim. Out of the 67 student arguments, 14 arguments included at least one *challenge*. Two challenges came from the teacher and students made 18 challenges. The challenges were mostly directed at property of operation claims that dealt with properties of addition and multiplication.

DISCUSSION

From the data, I identified elements of fourth-grade students' mathematical argumentation (claim, justification, qualifier, modification, challenge, background information, and examples). By characterizing the elements of an argument, I was able to elaborate on arguments at the fourth-grade level and develop a more field-specific model. Particularly, I elaborated on the current understanding of the purpose of examples and their use as either justification or background information. Past research studies on students' justifications, at different levels of school, have highlighted the use of empirical data as a way to convince (Carpenter, Franke, & Levi, 2003; Ball, Lewis, & Thames, 2008; Keith, 2006; Chazan, 1993; Harel & Sowder, 1998). I found that the

students not only use examples as justification, but also as background information when stating a claim. This finding contributes to the research literature by broadening what we understand to be the students' use of examples. There is evidence that students use examples in a role other than justification.

Claims are an important component of arguments (Toulmin, 1958; Ball, Lewis & Thames, 2008) and of this model of argumentation. Researchers have known that students in elementary school can propose claims and conjectures (Keith, 2006; Ball et al. 2008, Schifter et al., 2008), but the findings from this study add to the research by describing the claims that students develop more specifically. I identified two types of claims in the analysis of the data: (a) properties of operations and (b) classifications or definitions.

CONCLUSION

The empirical model adds to the research literature about upper-elementary students' arguments by detailing the elements of mathematical arguments and the roles the elements play in an argument at this grade level. Two of the analytical tools that already existed in the literature were used as a basis for this revised, more specific model. Research has shown that students use examples in their justifications (Chazan, 1993, Harel & Sowder, 1998, Carpenter, Franke, & Levi 2003), and the empirical model extends research on models of students' arguments by incorporating a place for examples and specifying the role that the examples play. Using the revised model I could represent an argument as it occurred between more than one person. I wanted to model a dynamic argument because arguments are naturally a social endeavor and that was not apparent in Toulmin's universal model of argumentation. Using the model, I was also able to look at specific elements of argumentation, characterize them, and explain how they were used by fourth-graders. The model was helpful in describing where fourth-grade students with limited exposure to argumentation begin. Even without prior instruction emphasizing argumentation, students bring relevant knowledge regarding arguments to the discussion that can be used to help students transition to more formal proof. A recommendation for future research includes trying to characterize student arguments with the model with other mathematical concepts and grade levels. Finding data that does not match the model, adapting the model, and the application of the revised model have the potential to strengthen the model and extend the research literature on upper-elementary school students' mathematical argumentation.

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