DEFINING THE NEED FOR JUSTIFICATION IN PROCESSES OF CONSTRUCTING JUSTIFICATIONS

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In the present research, we focus on processes of constructing justifications. According to Abstraction in Context, no constructing process will be initiated without a need (of the learner) for a new construct. In the current research we take a first step toward investigating students' need for justification by giving a definition for one aspect of this need and show the efficiency of that definition by analysing a case study of the role of this need when students construct a justification.

INTRODUCTION

Justification and Proof

Justification and proof are major components of mathematical reasoning and learning. Harel and Sowder (1998, 2007) defined proving as: "the process employed by an individual (or community) to remove doubts about the truth of an assertion" (Harel and Sowder, 2007, p. 6). This process contains two sub-processes: Ascertaining (in which individuals remove their own doubts) and persuading (in which individuals remove others' doubts). Harel and Sowder pointed out that they used 'proving' with the wider meaning of justification.

Abstraction in context (AiC)

Hershkowitz, Schwarz and Dreyfus (Hershkowitz, Schwarz, and Dreyfus, 2001; Dreyfus, Hershkowitz, and Schwarz, 2001; Schwarz, Dreyfus, and Hershkowitz, 2009) proposed the theoretical framework of Abstraction in Context (AiC). According to them, abstraction is an activity of vertically reorganising previously constructed mathematical knowledge into a new structure (Schwarz et al., 2009, p. 24).

According to AiC, in a process of abstraction the learner passes through three stages: the arising of the need for a new construct, the emergence of the new construct, and its consolidation. A central component of the framework is a model, which provides micro-analytic tools for exploring constructing processes. The model is based on three epistemic actions (Hershkowitz et al., 2001): Recognizing - in which the learner recognizes a previous construct and realizes that this construct is relevant to the problem presently at hand; Building-with - in which the learner acts with recognized constructs in order to achieve a goal such as solving a problem; and Constructing, the central epistemic action of the model, which consists of assembling and integrating previous constructs to produce a new construct. Constructing refers to the first time a new construct is expressed or used by the learner. One reason for using a model of epistemic actions is that these actions are observable. A reason for choosing these specific actions is that they have been found suitable and useful for investigating processes of abstraction.

2013. In Lindmeier, A. M. & Heinze, A. (Eds.). *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 1-8. Kiel, Germany: PME.

Constructing Justifications

Kidron and Dreyfus (2006, 2010) analysed a process of constructing justifications. They found that in a process of justification, several constructing processes may occur in parallel and interact.

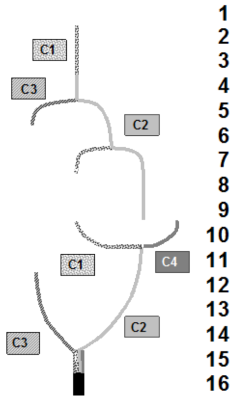

Figure 1: The interacting parallel constructions diagram

Figure 1 presents such a process of justification schematically. It shows four constructing processes marked as C1 - C4. In episode 5 construction C3 branches off from construction C2. In episode 11 constructions C1 and C4 combine. In the current study we will exhibit another case of a process of justification in which constructing actions combine.

Need for new construct and need for justification

Although AiC has been quite extensively used during the last two decades, research studies have focused far more on the emergence and consolidation stages than on the first stage: The need for a new construct, as well as the need for justification, has hardly been investigated. Schwarz et al. (2009) claimed that without a need for a new construct, there no construction is expected. The term "need" refers to the need of learners that occurs when their constructs are not sufficient for them in order to deal with a given mathematically situation. In the case of constructing justifications several kinds of need may occur. Kidron and Dreyfus (2006, 2010) observed a need for justification that refers to the need for achieving a deeper understanding of the sense of a statement. Kidron, Bikner-Ahsbahs, Cramer, Dreyfus and Gilboa (2010) proposed the notion of General Epistemic Need (GEN), namely a need to make progress in the constructing process. In the current study, we are using Harel and Sowder's (1998, 2007) definition for justification in order to explore still another aspect of the need for justification. Since they defined justification as a process employed by an individual to remove doubts about the truth of an assertion, we consider the need for justification as the need to remove doubts about the truth of an assertion. In the present research we demonstrate and explain this kind of need by means of the analysis of a case study.

METHODOLOGICAL CONSIDERATIONS

The current research is part of a wider study in which we examine the role of the need for justification in processes of constructing justifications. For this purpose we designed ten activities. Each activity is being carried out by at least three pairs of students. The activities are carried out as task-based interviews by pairs of students. For each activity, we conducted an a priori analysis, in which we attempted to determine the elements of knowledge that are expected to be necessary or useful to complete the activity, as well as the connections between these elements of knowledge.

Population and procedures

Here, we present a case study of one pair of students, Hadar and Shaked. The participants were grade 12 students from a high school in Israel. They were chosen from the advanced mathematics stream since they needed to deal with a mathematical situation involving integrals.

The task

The task given to the students presents two dilemmas. In the first dilemma the students have to decide whether the following claim is true:

In domains in which the function *f(x)* takes on positive / negative values, the graph of its anti-derivative increases / decreases.

In the second dilemma the students have to decide whether the following claim is true:

In domains in which the function *f(x)* takes on positive / negative values, the computation of the integral: $\int_a^b f(x)dx = F(b) - F(a)$ for $a < b$ yields a positive / negative result.

The dilemmas were situated in a story about a teacher who drew two graphs (see Figure 2) and told her class that the drawing contains the graphs of a function and of its anti-derivative.

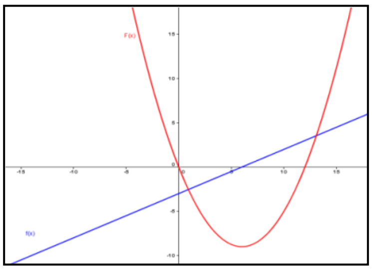

Figure 2: The graphs used in the activity

The story describes two students' controversy about the validity of the claims for all (continuous) functions. This is expected to raise doubts and hence to evoke the participants' uncertainty and need for justification.

A priori analysis

The analysis below will focus on the second dilemma. In the a priori analysis for the justification process relating to the second dilemma, we predicted the following elements of knowledge as relevant:

E1: In domains in which a function is positive / negative, its anti-derivative function increases / decreases.

E2: In domains in which a function increases / decreases, for $a > b$ the point (a, $f(a)$) is higher / lower than the point ($b, f(b)$).

E3: The value of the integral over f from a to b is given by the expression: $\int_a^b f(x)dx = F(b) - F(a)$.

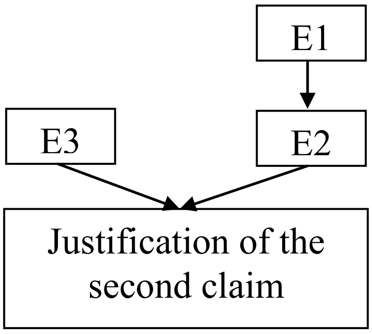

Figure 3: The connections between elements of knowledge

Our working hypothesis that the participants had constructed the three elements before attempting the second dilemma; for example, they had constructed E1 when dealing with the first dilemma. Figure 3 presents our a priori analysis of the connections to be established between these elements of knowledge during the justification process.

ANALYSIS OF THE CONSTRUCTING PROCESS

Hadar and Shaked had no difficulties figuring out that the first claim is true. For example, Hadar (H) explained as follows:

30	H:	The graph of f(x) represents the graph of the derivate of the anti-derivate function... and domains in which the values of the derivate are positive, the function increases.

We will focus on Hadar's process of constructing the justification of the second claim (lines 67 – 214) since she expressed herself clearly. We divided the relevant excerpt into segments, and analysed it as follows (H denotes Hadar, S Shaked, and I the interviewer):

Segment 67-73 contains the first reactions of the students to the second dilemma.

67	S & H:	Silence (20 seconds)
68	I:	Hadar, did you understand what she claimed?
69	H:	I did. But I want to figure out whether it is true or not.
70	I:	What did she say?
71	H:	She said that if the values of the function are negative, the anti-derivative of that specific domain will also be negative. Like if I think of the integral as representing the area, so the function… Like the area underneath the function in that specific domain. So if the function is negative the area…
72	S:	Supposed to be also negative.
73	H:	Yes… but no. I am trying to find a contradictory example. I am not sure if it is true.

In lines 69 and 73, Hadar expressed doubts about the claim: "I am not sure if it is true". Moreover, she is actively trying to remove the doubts: "I am trying to find a contradictory example". According to our definition, Hadar exhibits a need for justification that the claim is correct. This is the first expression of Hadar's doubts; it exposed her need for justification and led to the first stage of constructing the justification.

In segment 76 – 84 Hadar makes her first steps in constructing the justification.

76	H:	O.K if the calculation leads to a positive result then *f(b)* is supposed to be greater than *f(a)* (points to $\int_a^b f(x)dx = F(b) - F(a)$). Like, according to simple mathematics.

In line 76 Hadar recognized (R) the relevance of the expression $\int_a^b f(x)dx = F(b) - F(a)$.

In segment 85 – 87 Hadar continues trying

85	H:	And if this is negative (points to $\int_a^b f(x)dx$) so that (points to $F(b)$) less than this (points to $F(a)$). Now… If the function gets negative values… may I draw here?
86	I:	You may draw whatever you want. You got there some paper.
87a	H:	Let's say I will do… like in the beginning (she sketches a graph).
87b	H:	O.K it is declining which means that this (points to some point of the anti-derivative function in Figure 2) will be higher than this (points to a lower point on the anti derivative) did you understand what I am talking about?

In line 85 Hadar tried to figure out the connection between the value of the expression: $\int_a^b f(x)dx = F(b) - F(a)$ and the domains of negativity of the function *f*: She is trying to build with (B) the element of knowledge E3 a small step toward the justification.

In line 87a Hadar pointed out the connection between the negativity of the graph of the function and the decline of the anti-derivative function. By that she recognized (R) the relevance of the element of knowledge E1 for the process of the justification. If we relate it directly to the constructing process that began in line 85, we come to the conclusion that Hadar is trying to construct the connection between E1 to E3. In line 87b Hadar examined the meaning of the decrease of the anti-derivative function. When doing that, she mentioned the element of knowledge E2. We point out that in the current segment Hadar mentioned all three relevant elements of knowledge. Furthermore, while building with (B) those elements, she constructed the connection between E1 and E2.

In segment 88-94 Hadar made another step. We focus on line 89:

89	H:	If this is negative (pointing to the function *f*(*x*) in Figure 2) so that function (pointing to the anti-derivative function) decreases. Which means that *b*… but *b* comes before *a*. No? ...

In line 89 Hadar expressed the need to remove the uncertainty about the question: where should she place the parameter *b* (of the expression: $\int_a^b f(x)dx = F(b) - F(a)$).

Therefore, she expressed a need for justification. This is the second expression of Hadar's need for justification. While the need for justification in segment 67-73 referred to the main claim, in the current segment the need for justification refers to a sub-assertion that arises while trying to construct the justification for the main claim. In the current segment, Hadar is trying to combine between the elements of knowledge E2 and E3. As can be seen from the a priori analysis (Figure 2), in order to construct the justification, Hadar needs to combine those elements of knowledge. Therefore, the expression of Hadar's doubts indicated her need for constructing and combining elements that were required for the process of constructing.

In segment 95-100 Hadar asked the interviewer for confirmation that she is on the right path:

95	H:	Are we doing O.K?
96	I:	I really don't know.
97	H:	seriously?
98	I:	No.
99	H:	So, yes or no?

We interpret this segment as expression of Hadar's doubts and her need to remove them. Therefore we consider it as expressing a need for justification. In this segment Hadar's doubts reflect her need to find a way to justify the claim. This is her third expression of need and it shows Hadar's awareness that she has not yet finished the process of constructing the justification, and this in spite of the fact that she had already pointed out all elements of knowledge that needed for the justification. In other words, she was aware that she had not yet established all relevant links between these elements.

In segment 204-214 Hadar finished constructing the justification:

208	H:	O.K. domains in which the function is negative, it is negative, therefore, $F(a)$ will always be higher than $F(b)$ on the function.
210	H:	Because the function is decreasing.
210	H:	$F(b) - F(a)$ will be always negative. Because $F(a)$ will be always higher.

Here Hadar pointed out not only the relevant constructs but also made the appropriate connections between them. After this segment we haven't found any expressions of Hadar's need to remove doubts. Hadar's process of constructing the justification can be summarized as follow: First, she constructed the connection between E3 and E1 (segment 85-87). Then she constructed the connection between E1 and E2 (segment 85-87). After that, she tried to combine between E2 and E3 (segment 88-94). At the end she connected between all the three elements. Figure 4 presents schematically Hadar's process of constructing the justification.

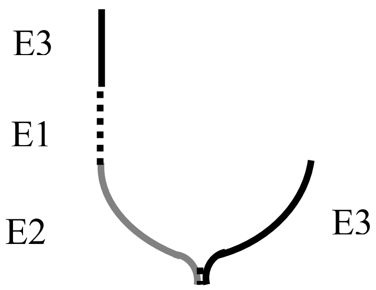

Figure 4: Hadar's construction process

CONCLUSIONS

In the current study we defined one aspect of the need for justification as the need to remove doubts. The analysis shows that this need can be detected from a student's utterances. Each of Hadar’s expressions of her need to remove doubts revealed a different stage of the need for the process of constructing the justification. The first indicated the need for constructing the justification, the second referred to the need for constructing and combining elements that were required for the process of constructing, and the third showed that the process had not yet come to its end. Hence we come to the conclusion that the need to remove doubts can be used for analysing the need for justification in processes of constructing justifications. As a result, it will be the basis of further research studies we plan.

ACKNOWLEDGEMENT

We would like to express our gratitude to Prof. T. Dreyfus from Tel Aviv University whose help, and stimulating suggestions helped us writing this paper.

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