EXTENDING THE COORDINATION OF COGNITIVE AND SOCIAL PERSPECTIVES

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Cognitive analyses are typically used to study individuals, whereas social analyses are typically used to study groups. In this article, I make a distinction between what one is looking with (one's theoretical lens) and what one is looking at (e.g., an individual or a group). By emphasizing the former, I discuss social analyses of individuals and cognitive analyses of groups, additional analyses that can enhance mathematics education research. I give examples of each and raise questions about the appropriateness of such analyses.

In recent years, there have been a number of theories of learning available in mathematics education (Lerman & Tsatsaroni, 2004). The advantages of multiple theories in mathematics education research can be realized through a diverse set of research projects generated and structured using different theories.

These advantages can also be realized through coordination of two or more theoretical perspectives in a single project when appropriate (e.g., Cobb & Yackel, 1996). In the case of cognitive and social theories, researchers have pointed to the complementarity of the theories (Cole and Wertsch, 1996; Kieren, 2000; Niss, 2006) and argued that each theory has its affordances and limitations that make it the tool of choice for some kinds of work and less useful for others (Simon, 2009).

In this article, I present a perspective on the domain of utility of cognitive and social perspectives that has emerged in my research projects and contrast it with a well-known existing theory. My hope is that this article will spark ongoing conversation about the issues raised and stimulate efforts to refine the ideas presented.

One of the better-developed examples of combining two theoretical approaches is the emergent perspective (c.f., Cobb and Bauersfeld, 1995; Cobb & Yackel, 1996). The theory is based on the coordination of a cognitive and a social perspective. It was developed for the purpose of characterizing mathematics learning in classroom settings. A social perspective is used for characterizing learning when the unit of analysis is the class (including the teacher). A cognitive perspective is used when the unit of analysis is individual students.

The use of different theoretical tools for analysis of these different units of analysis gives the theory a certain elegance and clarity. The analysis of classroom observations is done by looking at emerging norms (social and socio mathematical) and the identification of a sequence of mathematical practices developed over time. These analyses are coordinated with data from interviews with individual students. The interview data is analysed using a constructivist perspective, identifying the

2010. In Pinto, M. F. & Kawasaki, T.F. (Eds.). *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 177-183. Belo Horizonte, Brazil: PME.

students' conceptualizations at different points in the classroom design experiments (Cobb, 2003).

At a certain point in the evolution of our research program, my colleagues and I attempted to use the emergent perspective for careful examination of learning in classrooms, but found it inadequate for our purposes without modification. The problem was the following.

In the work of Cobb and colleagues (e.g., Bowers, Cobb, & McClain, 1999), the characterization of learning in a mathematics class results in postulating a set of mathematical practices. Typically, learning over the course of an academic year is characterized by a small set of mathematical practices. These practices, which may take weeks or months to develop, are too gross a tool for the detailed work we were attempting to do. We were interested in understanding the process that takes place as students move from one mathematical practice to the next.

What theoretical tools might we use for this purpose?

In bringing theory to bear on this problem, we began to think about theories of learning using the following distinction.

We distinguished what we are looking *with* from what we are looking *at*.

This distinction allowed us to go beyond the use of social theories for studying classroom data and cognitive theories for the study of individual data. Rather we thought about our work using the 2x2 matrix in Figure 1.

The upper left quadrant and the lower right quadrant present no problems. They are characteristic of the emergent perspective as well as many other research programs that fall into one of these quadrants or both. That is, it is commonplace for researchers to conduct cognitive analyses (e.g., constructivism) of individuals' mathematical thinking and social analyses of mathematical communication in small groups and whole-class discussion (e.g., socio cultural theory, symbolic interactionism).

However the upper right and lower left quadrants merit discussion.

Do these quadrants represent valid types of analysis? Let us consider each in turn.

SOCIAL ANALYSIS OF AN INDIVIDUAL

This quadrant represents work that socio cultural theory has been doing for some time, social analysis of an individual (e.g., a student working on a problem in isolation. In analyses characterized by this quadrant, the researcher considers that the activity of a child working by herself is influenced by the norms and practices of her mathematics class, the language that she speaks, the cultural practices of her family, etc. Such social explanations can be useful in understanding the activity and learning of the individual.

One might argue against such an analysis by asserting that no matter what influences the social environment exerts on the individual, the results of those influences are reflected in the individual's cognition. Even if one buys this argument, it does not negate the value of a social analysis of an individual engaged with a mathematical task in terms of the data that are considered and the constructs one can bring to accounting for those data.

Again, this point is based on the distinction between *looking with* and *looking at*.

Analysis	**Cognitive**	**Social**
Individual	cognitive analysis of individual	social analysis of individual
Group	cognitive analysis of group	social analysis of group

Figure 1: Analysis categorized by both the nature of the analysis and by the subject of the analysis.

Let us consider an example.

A student is working alone on the following question provided by the researchers: "What is the purpose of the multiplication step in the traditional long division algorithm?"[1]

Posing the question might be intended to find out whether the student understands the steps of the algorithm. Evidence of such understanding might be an answer of "To determine how many of the original set of items have already been put into groups." Instead, the student responds, "To find out whether the number that you put up top [in the quotient] is too large." Rather than making a cognitive interpretation of these data (a conjecture about the student's understanding of division), the researchers might consider a social explanation. They might conjecture that the student's response reflects his participation in a mathematics class in which the procedural role of algorithmic steps is emphasized (valued).

Thus, the student interprets the question as pertaining to the role of the multiplication step in obtaining a correct answer. Without looking with a social perspective, this interpretation of the data might not be considered.

[1] This refers to the algorithm most commonly used in the United States. For example:

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    38
3| |114
   -9
    24
   -24
     0
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COGNITIVE ANALYSIS OF A GROUP

Returning to our 2x2 matrix, the lower left quadrant, using a cognitive lens to focus on a group, is likely to be the most controversial.

Whereas, for the upper right quadrant, readers might readily accept that there is always a larger social frame for individual thought and action, applying a cognitive lens when looking at a group may seem less appropriate.

However, the argument for cognitive analysis of group activity, including discourse, is parallel to the one made for social analysis of individual action; it uses useful knowledge and constructs to expand what is noticed (what is identified as relevant data) and to generate useful explanations for the data. In attempting to do detailed analysis of learning in classrooms, we began to do analyses of this type.

As indicated earlier, social analysis of classroom learning using the theoretical construct of mathematical practices did not allow for the detailed distinctions that we needed to make in our work. However, a vast knowledge base of distinctions about student conceptions exists in the context of cognitively oriented theories. We continued to use the social aspect of the emergent perspective to analyse classroom norms (i.e., the conditions for learning in the classroom).

However we used constructivist analyses to understand the learning processes, making use of the rich empirical results of prior work on individuals' conceptions (c.f., Simon, Tzur, Heinz, Kinzel, & Smith, 2000; Simon & Blume, 1996).

Whereas from a social perspective, a conversation might be seen as a negotiation of meaning or increasing participation in the practices of the group, it has proved advantageous to also view it as two (or more) students with different conceptions attempting to understand the ideas of the other(s). At other times, it has been helpful to characterize the current "conception of the class" in relation to the conceptual goal of the teacher.

Let us consider an example.

In one of our classroom teaching experiments (Simon & Blume, 1994), students were asked to find the number of non-square, cardboard rectangles (of which they had a sample) that could fit on their rectangular table. They did so by using the rectangle to measure along the width and the length of the table and multiplying the two measurements.

From an observer's perspective, the students were finding the area of the table measured in cardboard rectangles of the given size. The instructor then asked the students to consider a solution in which a hypothetical student measured the width and length of the table, each time using the *long side* of the non-square cardboard rectangle (a strategy that was inappropriate for the original task of finding the number of rectangles that could fit on the table). The instructor asked about the meaning of the product of these measures. The consensus response of the students was that the number did not represent anything meaningful. Simon and Blume engaged in a cognitive analysis of the ensuing classroom discussion. Using these and other related data, the research team proceeded to develop a hypothesis regarding the students' understanding and its implications with respect to the development of multiplicative units. (See Simon & Blume, 1994, for a detailed analysis.).

The use of prior research on students' conceptions and the constructivist-based analysis of individual contributions to the conversation led to this hypothesis. For example, consider the following claim from the study which built on Thompson's (1994) work on quantitative reasoning: "Our analysis has resulted in a hypothesis that the [learner's] anticipation of the structure of the quantified area (a rectangular array of equivalent units) is a first step in the quantitative reasoning involved in evaluating the area of a rectangle" (Simon & Blume, 1994, p. 492). The cognitive analysis of the data from the whole-class discussion afforded the kind of detailed analysis that fit our objectives.

The analyses that we conduct that fit into this quadrant, while generative, do not come without some cost. They do not have the elegance of the emergent perspective, which uses different theoretical constructs for different units of analysis. The developers of the emergent analysis were careful to make sure that there was a fit between the analytical unit and the nature of the claim (e.g., a class unit can have an established mathematical practice, but not a concept).

Does the lack of this type of clarity make our extension of the use of constructivist theoretical constructs unwarranted? One thought is that the researcher is like an orchestra conductor, at times she is conducting the orchestra as a whole and at times she is concerned with what the parts are doing differently. Each of these activities is not discretely set off form the other. This problem is one that requires discussion and ongoing work.

CONCLUSION

I used the 2x2 matrix to highlight two types of analyses that are not typically discussed in mathematics education research, cognitive analysis of data involving a group and social analysis of data involving an individual acting alone.

The argument for such analyses is based on the notion that *cognitive* and *social* refer to the theoretical constructs that the researchers use to structure their observations (identification of relevant data) and to account for those observations.

This emphasis on what the researcher is looking *with*, as opposed to what the researcher is looking *at,* is aimed at maximizing the constructs available for data collection and analysis. Labinowicz (1985) pointed out, "We see what we understand." The attempts to expand theory use described above are aimed at using as much of our understanding as possible in analysing mathematics learning situations. In both individual and group situations, cognitive and social constructs can provide tools for research analysis.

Having made the distinction between what the researcher is looking *with* and what the researcher is looking *at*, I now must add the caveat that these two categories are not always independent. This is probably the reason that most analyses are focused on the upper left and lower right quadrants.

The purpose of studying an individual may indicate the use of particular cognitive constructs and the purpose of studying a group may indicate the use of particular social constructs.

Conducting a rigorous analysis using the class as the unit of analysis (as opposed to studying the interacting parts) has particular affordances.

Whereas these points are important to recognize, the purpose of this article is to call attention to situations in which one can make greater use of theoretical constructs and empirical results that are available to mathematics education researchers.

Further, it raises questions about the proposed ways of doing so.

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