PROOF COMPREHENSION AT THE UNDERGRADUATE LEVEL

Juan Pablo Mejia-Ramos [1], Keith Weber [1], Evan Fuller [2],
Aron Samkoff [1], Kathryn Rhoads [1], Robert Search [3]

[1] Rutgers University, [2] Montclair State University, [3] Centenary College

Although proof comprehension is fundamental in higher-level undergraduate mathematical courses, there has been no research on what it means to understand a mathematical proof at this level and how such understanding can be assessed. In this poster we address these issues by presenting a multi-dimensional model of proof comprehension and illustrating how each of these dimensions can be assessed.

To identify the ways in which a proof might be understood at the undergraduate level, we took into account a wide variety of sources, including theoretical articles on how proofs are used by the mathematical community, empirical studies investigating mathematicians' behaviors when reading proofs, and reading comprehension models. Building on Yang and Lin's (2008) model of reading comprehension of proofs in high school geometry, we argue that in undergraduate mathematics, a proof is not only understood in terms of the meaning, operational status and logical chaining of its statements (as Yang and Lin delineate), but also in terms of its higher-level ideas, the methods it employs, or how its steps can be illustrated with specific examples. Accordingly, we outline six ways a proof may be understood at the university level:

1) *Meaning of terms and statements*: Understanding the meaning of the theorem and other statements, terms, and definitions in the proof;
2) *Justification of claims:* Explaining how each assertion in the proof is justified;
3) *Logical Structure:* Identifying the proof's overarching logical framework;
4) *Higher-level ideas:* Compartmentalizing the proof into chained modules and identifying its main ideas;
5) *General method:* Identifying the techniques used in the proof and the ways in which these procedures can be used to approach other proving tasks; and
6) *Illustration with examples:* Understanding how the proof relates to and could be illustrated by specific examples or graphical representations.

The poster presents this multi-dimensional model in a graphical format, illustrating a two-column parallel of ways in which each of these types of understanding may be assessed in the context of two specific proofs. Further, a third column presents template questions that lecturers and researchers can easily adapt to their own proofs.

References

Yang, K.-L., & Lin, F.-L. (2008). A model of reading comprehension of geometry proof. *Educational Studies in Mathematics, 67*, 59-76.

2010. In Pinto, M.M.F. & Kawasaki, T.F. (Eds.). *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 365. BeloHorizonte, Brazil: PME.