BUILDING UNDERSTANDING THROUGH PROOFS

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The importance of proofs is well established in the mathematics education. However, there is no consensus on the best ways to promote them mathematics classrooms. Some educators insist on formal deductive proofs, but a growing number of educators emphasize *heuristics*, *reasoning* and *justification.* They argue that investigative and problem-solving skills are more "useful" and "enjoyable," and more of a "human activity" than formal proofs (Simon & Blume, 1996). The availability of dynamic software such as Geometers' Sketchpad and Cabri Geometry is regarded as partly responsible for this approach (Hanna, 2000). However, there is agreement that proofs must promote mathematical understanding (ICMI-19 Study). This paper will report the work of five high school students on a challenging probability task. In particular, the paper will describe how the students' attempts to prove their solution allowed them to gain a deeper understanding of the mathematical ideas involved in the task. The data comes from the database of a 16-Year NSF-funded longitudinal study on students' development of mathematical ideas in problem-solving contexts. The probability task in question stated that two teams played each other in at least four and at most seven games. The first team to win four games is the winner of the World Series. Assuming that the teams are equally matched, and there are not ties, the students were asked to compute the probability that the World Series will being won in a) in four games b) in five games c) in six games d) in seven games. The students worked on the problem in three 2.5-hour occasions over a six-month period. The students were asked to not only come with a solution but also to provide convincing justification for the solution. They were even asked to compare their solution to an "incorrect" one by a group of graduate students. The Analysis of the students' mathematical activity on the probability task revealed rich evidence of growth in the students' understanding of mathematical ideas involved in the task. In particular, the students built powerful isomorphism with pas tasks, articulated elegantly the relation between the task and Pascal Triangle, and deepened their understanding of importance of [un] equally likely events in determining probabilities. The results are particularly significant since the students had limited exposure to formal probability.

References

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