STUDYING TEACHERS' PEDAGOGICAL ARGUMENTATION

Metaxas Nikolaos, Potari Despina, Zachariades Theodossios

Department of Mathematics, University of Athens, Greece

We present a case study analysis of the arguments used by an experienced high-school teacher. We employ the model of argumentation schemes in adjunction with Toulmin's scheme. We focus on the content and the structure of the arguments used, and examine the different aspects of teacher knowledge that emerge. There is evidence of a structured example space which instantiates through the use of pedagogical examples as a conclusive and integral part of teacher's argumentation. This example space is based on a sense of deep pedagogical intuition framed by teacher's craft knowledge.

INTRODUCTION

There is a growing amount of research in mathematics education literature focusing on mathematical arguments (structures of inference with mathematical meaning) produced by students and teachers of mathematics. The main methodological tools for the analysis of these arguments are primarily Toulmin's model (Toulmin, 2003). Some researchers use Toulmin's model as a lens through which they document students' learning progresses in a classroom (Krummheuer 1995), while other researchers study the quality of a certain mathematical argument (Pedemonte, 2007). Nevertheless not much has been done in the direction of analysing the pedagogical arguments of teachers. In this paper we use Toulmin's model coupled with argumentation scheme analysis (Walton and Reed, 2005; Walton, Reed & Macagno, 2008) to dissect the structure of argumentation of mathematics teachers when they interpret and comment on students' answers and design teaching interventions to help students overcome emerged difficulties. By examining the kind of arguments they employ and studying their structure we seek to investigate some aspects of teacher knowledge (Schulman, 1987; Ball, Thames & Phelps, 2007) that is lying beneath each argumentative scheme. So we examine the following questions: (a) what is the kind of arguments teachers use, (b) what is the structure of these arguments and (c) what can be inferred about teachers' knowledge.

THEORETICAL PERSPECTIVE

Toulmin's (2003) model asserts that most arguments consist of six basic parts, each of which plays a different role in an argument. The claim (C) is the position or claim being argued for. The data (D) are the foundation or supporting evidence on which the argument is based. The warrant (W) is the principle, provision or chain of reasoning that connects the data to the claim. Warrants operate at a higher level of generality than a claim or reason, and they are not normally explicit. The Backing (B)

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provides the support, justification or reasons to back up the warrant by presenting further evidence. The modal qualifier (Q) represents the verbalization of the relative strength of an argument and the rebuttal (R) consists of exceptions to the claim stating the conditions under which it would not hold. Since Krummheuer (1995) a lot of research has been taken place using Toulmin's scheme. Nevertheless there are some notable difficulties. The notion of warrant has proved notoriously difficult to interpret, while due to the elliptic form of human argumentation not all the premises are explicitly stated. In order to bypass such turns, we turn to a useful tool in argumentation theory: the theory of argumentation schemes. These are forms of argument that represent structures of common types of arguments used in everyday discourse as well as in special contexts like those of scientific and legal argumentation . Their flexibility to accommodate deductive, inductive and abductive (or defeasible) forms of arguments has led to a recent paradigm shift in logic, artificial intelligence and cognitive science. Recent work in analysis of presumptive argumentation and argumentation schemes (Walton et al.; 2008) has led to an extensive compendium of argumentation schemes on which we base our work in the classification of the arguments used by mathematics teachers. The structure and the content of argumentation schemes can reveal facets of teachers' pedagogical content knowledge (PCK). We use the term PCK in the sense of Schulman (1987) as refined by Ball et al (2007) who proposed a further classification of content knowledge for teaching. Two basic components of this classification are i) PCK, which contains the subdomains of knowledge of content and students (KCS, is knowledge that combines knowing about students and knowing about mathematics) and knowledge of content and teaching (KCT, is knowledge that combines knowing about teaching and knowing about mathematics), and ii) specialized content knowledge (SCK) which is the mathematical knowledge and skill uniquely needed by teachers in the conduct of their work and belongs to the general domain of content knowledge. Finding an example to make a specific mathematical point is one of the mathematical tasks of teaching that characterize SCK and mirror a teacher's conceptions of mathematical objects involved in an example generation task, his pedagogical repertoire, his difficulties and possible inadequacies in his perceptions (Zazkis & Leikin, 2007). We examine the examples that teachers provide in order to support an argument, as an indicator of their mathematical and pedagogical knowledge. We employ the notion of example space by Watson and Mason (2005) which is a collection of examples that fulfill a specific function influenced by individual's experience and memory, as well as by the specific requirements of an example generating task. In our case several argumentation schemes include as support examples of pedagogical issues. In order to differentiate them from mere mathematical examples or empirical examples (that are used as data of an argument) and to emphasize the metalevel they belong to relatively to the ground level of the didactic episode they talk about, we adopt the term metaexamples (Metaxas, 2008).

METHODOLOGY

This study is part of a larger research project investigating PCK and SCK that high school mathematics teachers have as well as the evolution of their knowledge bases. The participants were 18 high school mathematics teachers with a degree in mathematics and were enrolled in a 2-year master's program in mathematics education. In partial fulfilment of their master's degree, they had to attend thirteen 2-hour classes of a semester long graduate course in Didactics of Calculus and this study was based on this course.

Process: The process in tentative chronological order was as follows: i) Observation and video recording of the lessons of the course. The lessons were based on a number of tasks which contained hypothetical didactic situations (Biza, Nardi & Zachariades, 2007) and they triggered off extensive discussions on mathematical, pedagogical and didactical issues, while the role of the instructor (and member of the research project) was that of a facilitator. The class was scheduled to adhere to the basic complex science principles (Davis & Simmt, 2006) of decentralized control (a complex form is bottom-up; its emergence does not depend on central organizers or governing structures), neighbour interactions (the neighbours that must 'bump' against one another are warrants, ideas, and other parts of argumentation) and enabling constraints (The participants, for example, expect the topics of discussion to be appropriate to their work etc.). ii) A half an hour tutoring audio-taped session with a high school student. The student was given a worksheet with some calculus tasks designed by each participant teacher and in the tutoring session the teacher discussed with the student his/her responses. This happened during the first week of the classes and the same was repeated during last week. iii) Two semi-structured interviews with six of the participants. The interviews were based on teachers' answers to the tasks and on the analysis of their videotaped classes and audio taped tutoring sessions. The two interviews, approximately 90 minutes each, were held during the first and last week of the semester. The purpose of the questions addressed to the teachers was to make them elaborate on their written answers and oral arguments they used in class and unveil their argumentation base. iv) Transcription and analysis of all video and audio tapes. v) Triangulation of the initial results obtained, by means of a third interview with every one of the six teachers where they commented on our own interpretations.

Data and data analysis: Data consisted of the: i) teachers' responses on the tasks used in the class, ii) videotapes of all classes, iii) two audio-taped half hour tutoring sessions of each participant with a high school student of his choice, iv) two semi-structured audio-taped interviews with each participant. We analysed line by line the dialogues, coded the parts of every argument as D,C,W,B,R or Q (the elements of Toulmin's model) and categorized them using a levelling structure similar to analysis by Chinn and Anderson (1998) : we consider wherever applicable, a Backing as simply a Datum in a second argument frame whose Claim is the Warrant (either explicit or entailed) from the first argument frame. This presupposes the existence of

a conceptual continuity among the arguments, which allows including them in the same unit, which we call it, a chain. A chain is a series of argumentation schemes that are linked with each other by a claim, a warrant, a backing or a rebuttal and share a common idea or concept in a gradually expanding web of interlocking argument frames. Each argumentation scheme is characterized by the level of depth it has in the chain (first level for the first scheme in order of a chain etc.). So for example, D/3 means a Datum in the third argumentation scheme of a certain chain. In order to study the kind and structure of inferences used, we characterized the arguments of each argumentation scheme according to a compendium of schemes (Walton et al., 2008). At the same time, we categorized each such scheme according to the kind of teachers' knowledge it exhibited (PCK, SCK, CKS, CKT). In order to check any discrepancy among our records we included a comparison of data collected in the video and audio tapes with the data from written records. We also allowed for a grounded approach: trying to take notice of any emerging pattern, either in the content or in the structure. Due to space limitations, we will present here the case of one of the teachers-participants, John (pseudo-name), with 20 years teaching experience of teaching calculus in high school. John was in the third semester of his graduate studies.

RESULTS

We will present some of the results based on the two interviews with John while including a characteristic extract from his first interview.

Types of arguments used

As it is clear from table 1 below, John mostly used five types of arguments while only in 4 out of 44 total cases he relied on a theoretical (pedagogical) argument with no support from experience. The main types of argumentation schemes (Walton et al., 2008) that John used were: *argument from illustration* (Premise 1: usually if x has property F the x has property G, Premise 2: in this case x has property F and G, Conclusion : the rule is valid), *argument from analogy* (Similarity Premise : generally case C is similar to case F, Base Premise : A is true in case C, Conclusion : A is true in case F), *argument from classification* (Premise 1: k has property F, Premise 2: for all x, if x has property F then x can be classified as having property G, Conclusion : k has property G), *abductive argument from effect to cause* (F is a set of facts in the form of an event that has occurred, E is a satisfactory explanation of F, therefore E is plausible as an hypothesis for the cause of F) and *argument from opposite* (the opposite of S has property P, therefore S has property not-P). The first three of the above kinds of arguments are basically inductive in their structure with one of their premises to be personal experience. This would probably mean that John was still relying heavily on intuitive knowledge he gained through his experience either as a teacher or as a high school student even after three semesters experience at graduate level. His extensive use of metaexamples (15 out of 19 argumentative chains contain metaexamples) concurs with the empirical backing that the above kind of arguments

usually have and help reducing the "cost" for understanding an argument (in the sense of Besnard and Hunter, 2008).

Number of arguments	Arguments from opposite	Arguments from analogy	Arguments from classification	Arguments from illustration	Arguments from effect to cause	General pedagogical arguments
46	2	22	6	10	2	4

Table 1: Classification of arguments used in two interviews

Structure of argumentation

The pattern we see also in the extract below, namely the conclusion of an argumentative chain by the means of a metaexample, is indicative of a more general trend : 13 out of 19 argumentative chains have a metaexample as a backing in the last argument of the chain. It signifies probably the conclusive character that his experience had for John, as he himself stated: "all the teaching episodes that I have experienced as a teacher or as a student, have strongly shaped my views and I regard them as a definite and valuable asset". On another aspect, considering the appearance of the modal qualifiers (Q and R) of the Toulmin's scheme, we note that 13 out of 14 of their total appearances are realized in arguments that are not backed by empirical data.

1	J(ohn)	What I do is writing certain examples on the board, so	D/1
2		I regard that a general solution method can be inferred from concrete	
3		examples….	C/1
4		For example, to check the continuity of a function I give some specific	
5		functions and discuss how to prove their continuity	B/1
6		Obviously, you can not see all the cases ….	R/1 & D/2
7		but still this teaching approach has better results	C/2
8		Otherwise, I think that describing to the students a general method from	
9		the very beginning you narrow their thinking….	Q/2 & W/2
10		the students can not think if constantly follow someone else's instructions	B/2 & D/3
11		Only after the examples we can generalize the method	C/3
12		….I think if the student discovers some rules on his own he understand	
13		better the mathematics behind them.	W/3
14		For example, in finding the number of the roots of an equation using	
15		Bolzano's theorem the student may realize the meaning of "at least one	
16		root" in the theorem statement	B/3 & D/4
17	R	Could this example be confusing for the not-so-good students?	
18	J	It depends on the classroom environment.	C/4

19	If classroom norms encourage the dialogue, a weak student won't hesitate	
20	to express his confusion whenever he feels so.	W/4
21	For example, I could ask them to construct a first order equation with no	
22	roots. Any student could feel free to ask for clarification.	B/4

In particular in the extract we cite, we see that in the first scheme John was stating his didactic method which he claimed it adheres to general pedagogical principle (line 2). An explicit warrant is missing, but he supplied as backing an example of a case in calculus (metaexample) (line 4) to illustrate his method. It's an argumentation scheme from analogy (Walton et al., 2008) with a premise based on his experience (so basically inductive). His rebuttal (line 6), which actually is an undercut of the claim (Besnard and Huntler, 2008) means that some uncertainty remains about the universality of his claim but (turning the previous undercut to a datum for a second level argument) he warranted his preferred method by stating another pedagogical argument and a qualifier is used to express the uncertainty of the validity of this argument (lines 8, 9). His 2nd level backing (line 10) now serves as a datum to a 3rd level argument. The warrants at levels 3 & 4 (lines 12,13 and 19,20) are again general pedagogical (student's initiated search and classroom norms) which are backed by metaexamples of teaching a certain calculus paragraph (lines 14-16 and 21-22). Both arguments are arguments from illustration (at level 3 it is coupled with a defeasible form of modus tollens).

Argumentation and teachers' knowledge

This extensive use of metaexamples in his argumentation schemes indicates the existence of a rich example space (in the sense of Watson and Mason, 2005) which is an indicator of familiarity with pedagogical issues like students' misconceptions, teaching methods etc. We concur with the view of Watson and Mason (2005) that "to understand mathematics means, among other things, to be familiar with conventional example spaces" and we note that in our case John's example space is pedagogical in nature and thus extends the above mentioned notion to the direction of pedagogical content knowledge. It is of significant importance that all the (meta)examples provided by John, were given by him spontaneously and as an integral part of his argumentation schemes. This means they are experienced as members of a structured space. Furthermore, the existence of a structured example space on the level of pedagogical knowledge shows a certain level of knowledge of the types of KCS and KCT. The conclusive character that his experience had for John as noted before, demonstrates a considerable degree of confidence in his empirical intuitions. This agrees with Fischbein's remark (1987) that overconfidence is related to the degree of intuitiveness of the various items considered and experience plays a fundamental role in shaping these intuitions either primary or secondary. The overconfidence is also indicated by the function of the modal quantifiers that we encounter. It augments the above mentioned note regarding metaexamples and empirical foundation and surely

casts a shadow on the strength of theoretical pedagogical knowledge that teachers - students are learning. This deep belief in his personal pedagogical experience without an equivalent abstract foundation is reminiscent of "deep intuition" of mathematics (Semadeni, 2008) that sometimes students may have without backing it with a sound theoretical knowledge. We could in a sense talk also here about a deep intuition of pedagogy (either as KCS or KCT) that doesn't rely on a deep theoretical knowledge. This deep intuition is a common characteristic among all the teachers that participated in this project.

Conclusion

In this paper, we have presented our preliminary findings emerging from our work on the study of argumentation of high-school math teachers. In particular, we analyzed the structure and the pedagogical content of their arguments. Methodologically, we used the theoretical framework of argumentation schemes extending Toulmin's model in order to obtain a more precise characterization of the arguments involved while with the notion of argumentation chain we took accountability of the intertwined nature of teachers' reasoning. In the case study of John, most of the arguments he used belonged to the "applying rules to cases" category (Walton et al., 2008) which are arguments that relate to a situation in which some sort of general rule is applied to the specifics of a given case. His backings in these argumentation schemes were mainly metaexamples while whenever he used modal qualifiers his premise was a general pedagogical rule, remark etc. It seems that a certain pattern emerges: he is overconfident about his personal empirical knowledge while he is quite reserved about general pedagogical statements. Furthermore, his accessibility to a wide range of metaexamples and his readiness to use them for supporting his arguments, indicates a strong pedagogical content knowledge (KCS and KCT) and a structured pedagogical personal example space that entail a deep pedagogical intuition. Also the place of them in the last part of an argument, shows again a belief in the concluding power of empirical knowledge. The suggested framework is a guideline, it is neither comprehensive nor complete but it offers a way to pose new questions that are related to teacher reasoning and knowledge.

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