THE IMPACT OF SUBJECT-SPECIFIC CURRICULUM MATERIALS ON THE TEACHING OF PROOF AND PROOF SCHEMES IN HIGH SCHOOL GEOMETRY CLASSROOMS

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This qualitative research study describes how three high school geometry teachers within the Midwest region of the United States used curriculum materials to teach concepts and skills associated with proof in high school geometry classes and the subsequent proof schemes exhibited by students within the classroom. Using a conceptual framework, data were analyzed on three dimensions: task features, cognitive demands of tasks, and the proof schemes employed in solving the tasks. The results suggest that external conviction proof schemes are more prominent in geometry classrooms when lower–level proof tasks are posed. The few instances in which higher cognitive demand proof tasks were posed, analytical proof schemes were much more prevalent.

Key words: proof, geometry teachers, curriculum materials

INTRODUCTION

The instructional responsibilities of geometry teachers are vast, and teaching proof is just one of them. Jones (2000) acknowledged that there are many things in the geometry curriculum that have to be taught so teachers have to consider: aspects of proofs that need to be accentuated, potential pedagogical strategies that can bridge deductive thinking and geometric insight, and tools that can facilitate students' learning to prove. Unfortunately, due to time constraints, teachers often choose to provide students with examples, and neglect to prove theorems (Fussell, 2005). Hence, efforts to improve students performance of proof by teaching it in creative ways have been ineffective based on the multiple research findings that suggests students continues to perform poorly on proof tasks (Battista & Clements, 1995; Healy & Hoyles, 2000; Recio & Godino, 2001; Senk, 1985).

Ball et al.(2002) identified three research areas that are needed pertinent to the teaching of proof: the refinement of the functions, perceptions and role of proof; empirical research documenting the challenges of learning to prove; and "the development, implementation, and evaluation of effective teaching strategies along with carefully designed learning environments that can foster the development of the ability to prove in a variety of levels…" (p. 908). This study aligns with the third primary need for research on the teaching of proof, because it seeks to examine how geometry teachers use curriculum materials in teaching proof. The need to gain a better perspective on how proof is taught and how teachers use curriculum materials to support the teaching of proof requires an examination of the differences between the written curriculum and the enacted curriculum with regard to proof. Hence, this study seeks to answer the overarching research question: How do teachers use curriculum materials to facilitate the teaching of proof in geometry? More specifically, what is the nature of the differences between how proof is presented in the written curriculum and how it is reflected in the enacted curriculum in a high school geometry course?

PERSPECTIVES

In mathematics courses, but particularly in geometry, "doing proofs" embodies various actions by teachers and students which are influenced by stated or implicit norms of what work is valued, the structure in which proofs ought to be presented, the time allocation for proofs, and the responsibility of students and teachers while "doing proofs" (Herbst et al., 2009). Teachers' responsibilities may include providing students with tasks, ensuring that sufficient details are provided (for example, the given and what needs to be proven) such that the students can attempt to construct proofs, activating students thinking by making suggestions about how to construct proofs, and ensuring reasons are provided for statements made (Herbst et al., 2009). Conversely, the students' responsibilities may include constructing proofs and marking the diagrams without altering them (Herbst et al., 2009). There is a reflexive relationship between students' perceptions of their role, the teacher's role, classroom social norms and what is deemed mathematical activity (Yackel & Cobb, 1996). Simon and Blume (1996) acknowledged that the class conceptualization of mathematics can affect their acceptance of correct justification. Since there exist variation in instructional practices for teaching proof (Harel & Sowder, 2007), which may impact students learning of proof (G. J. Stylianides, Stylianides, & Philippou, 2007), careful consideration must be given to the context and factors that influence how proof is taught and learned.

Teachers' beliefs and subject matter knowledge can significantly influence their classroom practices (Borko & Putnam, 1996). Hence, teachers' conceptions of proof may influence the way they teach it. Knuth (2002) conducted semi-structured interviews with 16 in-service high school mathematics teachers about their conceptions of proof in mathematics. He found that 75% of teachers considered that the role of proof was to communicate mathematics, and 50% of teachers considered that it was to systematize mathematical ideas and construct new knowledge. According to Stylianides, Stylianides and Phillippou (2007), "If teachers' knowledge of proof is fragile, that is, it is shaky and yields to attempts to inject contradictions into it … it is likely that teachers will teach proof poorly or will not teach proof at all" (p.146). Therefore, in examining how proof it is taught, it would be remiss to ignore the potential influence of teachers' beliefs on instructional decisions made.

Proof-related tasks in curriculum materials are often implemented as such during the enacted curriculum (Bieda, 2010). Bieda found that 71% of the tasks identified as proof-related tasks during the curriculum analysis were implemented as such by the teachers, 21% of the proof-related tasks were not implemented as proof-related task due to time constraints, and 8% of the proof-related tasks were not implemented because of teachers' discretion. Cirillo (2009) documented that a challenge in teaching authentic proof is that textbooks emphasize applications of theorems rather than their proofs. She recommended that greater emphasis be placed on the curriculum materials and objectives that teachers are given to facilitate the teaching of proof. Hence, it is important to consider not only the curriculum materials used, but also how they are used during instruction. Considering that curriculum materials are a major investment for school districts (Reys & Reys, 2006), and that proof has been identified as a core process of mathematics (NCTM, 2000), it would be beneficial to document how proof is bridged from the geometry textbook to the teacher, and from the teacher to the class.

METHOD

This study employed qualitative methods (case study research design) to investigate how three geometry teachers from the Midwest region of the United States used subject-specific curriculum materials (*McDougal Littell Geometry* (Larson, Boswell, Kanold, & Stiff, 2007) and *Prentice Hall Geometry* (Bass, Charles, Johnson, & Kennedy, 2004)) for teaching proof. The term "subject-specific" is used to differentiate the curricula studied from curricula where the content is integrated.

To ensure that teachers were familiar with the organizational structure of the curriculum materials, only teachers who had used the curriculum for at least 3 years were studied. The Mathematical Tasks Framework [MTF] (Henningsen & Stein, 1997) and a proof schemes framework comprises the conceptual analytical framework for this study. The MTF provide a means to analyze the level of cognitive demands of mathematical tasks written in curriculum materials, teachers enactment of mathematical tasks, and students' implementation of mathematical tasks. Harel and Sowder (1998) proof schemes framework considers the nature of the convincing arguments provided for proof tasks as enacted in the classroom. Harel and Sowder (1998) proof scheme classification includes three categories, which are not necessarily independent to each other: *external conviction, empirical* and *analytical*. Considering that the operational definition of proof in school mathematics (A. J. Stylianides, 2007) emphasized that "*Proof* is a *mathematical argument*, a connected sequence of assertions for or against a mathematical claim" (A. J. Stylianides, 2007, p. 291), it is important to consider the nature of the arguments deemed convincing. Hence, the analytic framework can be conceptualized as three-dimensional: the first two dimensions are drawn from MTF, and the last dimension utilizes Harel and Sowder's proof schemes. The first dimension represents the task features; the second dimension, the cognitive demands of the tasks; and the third dimension, the proof schemes emphasized to complete the tasks. Data were collected via video and audio-recorded classroom observations, teacher interviews, teacher artifacts, and textbook use diaries.

The data were analyzed using a six-stage analytical process. During the first stage a curriculum analysis was conducted using Henningsen and Stein's Mathematical Tasks Framework of tasks deemed to be proof or proof-related in chapters related to reasoning and proofs, parallel and perpendicular lines, and congruent triangles. Hence, the analysis focused on Chapters 2, 3, and 4 of Larson et al. (2007) *McDougal Littell Geometry* and Bass et al. (2004) *Prentice Hall Geometry* Teacher's Edition. In both books, Chapter 2 is entitled "Reasoning and Proof", and Chapter 3 and 4 is entitled "Perpendicular and Parallel Lines" and "Congruent Triangles" respectively. Among the tasks features coded, I considered whether the tasks were proof or proof-related, whether answers were provided with reasoning or as answers only; if the tasks were labeled “challenge”; if the tasks required one solution strategy or multiple solution strategies; whether the setting of the tasks were abstract, or in a realistic context; if the tasks required students to fill in the blanks; and if they were multiple choice or composed of multiple parts. The cognitive demand of the tasks ranged from *memorization* to *doing mathematics* (Henningsen & Stein, 1997). Proof tasks deemed as *memorization* generally required students to complete skeletal proofs in which they had to fill in the blanks, while proof tasks deemed as *procedures without connections* included matching statements with appropriate reasoning to complete a proof. Proof tasks classified as *procedures with connections* included items that required students to prove congruence of triangles in a Cartesian plane using a particular theorem, or write proof plans. I classified proof tasks as *doing mathematics* when tasks generally required students to write complete proofs. More particularly, to be coded as *doing mathematics*, a proof task required writing a complete proof, that was not similar to previous tasks and examples, and perhaps included changes to the context, utilizing a different representation, and was not algorithmic. During the second stage, the extent to which teachers used their textbooks to teach proof was summarized. For the third stage, proof-related tasks enacted in the classroom using the MTF were coded. For the fourth stage, factors influencing the set-up phase and students’ implementation during the enacted lesson were coded and classified. For the fifth stage, significant proof schemes used during the lesson were characterized. Proof schemes were coded based on the interactions between the teachers and students during the enacted lessons. At the sixth stage a profile of each case was constructed to represent the 3-dimensional conceptualized analytical framework, in an effort to conduct a within case analysis, as well as a cross case analysis (1998). Multiple researchers assisted with the analysis for reliability purposes.

RESULTS

The results suggest that most of the tasks within the subject-specific geometry textbooks, for Chapters 2-4, requires lower-levels of cognitive demand that encourages the use of *procedures without connections*. An analysis of proof tasks revealed differences between the textbooks task features and cognitive demand of proof tasks. For instance, of the 1066 tasks analyzed in *Prentice Hall Geometry* (Bass et al., 2004) only 79 of them were proof tasks; and almost half (46.8%) of the proofs required students to fill in the blanks. On the other hand, of the 977 tasks analyzed in *McDougal Littell Geometry* (Larson et al., 2007), 128 were proof tasks. Only 10.2% of the proofs in this textbook required students to complete skeletal proofs. Therefore, for the chapters studied, *McDougal Littell Geometry* provided more opportunities for students to write complete proofs than *Prentice Hall Geometry*. Nevertheless, despite the differences, proof tasks when enacted generally reflected lower-levels of cognitive demand.

The geometry textbooks significantly contributed to instructional methods as to how proof was taught, structure of lesson, and the proofs to which students were exposed for homework and practice assignments. Admittedly, in many instances students’ opportunity to prove required them to fill in the blanks or provide missing information for a statement and/or reason to make a complete proof. The tasks that are labeled "challenge" are at the end of the exercises and are seldom assigned.

Teachers' implementation of proof tasks reflected how it is presented in textbooks. During whole class discussions, the teachers generally completed the proofs for students and posed questions that required recollection of facts rather than the elaboration of arguments. Contrary to the similarities of teaching methods, the results of instruction varied because of students' disposition, and community of practice. Factors such as assessment, and teachers' desire to make mathematics "easy" also influenced how proof was taught.

Finally, it appears that there exist a relationship between the level of cognitive demand of a task and the proof scheme utilized. *External conviction proof schemes* were more visible, when *memorization* and *procedural tasks without connections* were posed. Teachers and textbooks were viewed as the authority of the mathematics, and students were encouraged to mirror the teacher's actions in constructing proofs, and recall the appropriate reason to fill in the blank from a list of reasons memorized. It appeared that teachers sought to make the learning of proof more "comfortable" or "easy" such that students attain some form of success. The emphasis on rules and memorized reasons from a prepared list might have impaired creativity and originality, since students at time randomly selected reasons without evaluating the appropriateness of their choices. The few instances in which *procedures with connections* proof tasks were posed, *analytical proof schemes* were evident. Whenever higher-level cognitive demand proof tasks were posed, students were challenged to logically link statements and reasons to create an acceptable proof, rather than merely focus on how many steps are needed to construct the proof. There was a single lesson in which *empirical proof schemes* was observed; and when *empirical proof schemes* was used, it was for a proof-related task that required *procedures with connections*.

IMPLICATIONS

Considering, the major role geometry textbooks plays in the teaching and learning of proof, the tasks posed need to be reviewed and revised in efforts to provide greater opportunity for students to create original proofs, and engage in proof activities that reflects the doing of mathematics. Currently, the geometry textbooks studied, limits the development of proof conceptions that can be enhanced by exploration or discovery by stating main ideas in the summary section, providing proof for postulates in the examples, or posing excessive amounts of proof tasks which only require students to fill in the blank or to identify the missing link in proof tasks.

The teachers' decision to make proof "easy" for students might have hindered students conceptualizing how to construct rich proofs. Hence, it is recommended that teachers who improvise and adapt the content of the geometry textbooks seek to challenge students to think critically, and learn how to construct proofs (beyond the realm of two column style proofs), rather than just encourage students to memorize a list of reasons and the structure of proofs.

Finally, the potential relationship between the level of cognitive demand of the tasks and the proof schemes suggest that teachers ought to seek to pose higher-level cognitive demand proofs if students are to attempt proof tasks using *empirical* or *analytical proof schemes*. This result reaffirms the need for textbooks to increase the percentage of higher-level proof tasks and/or proof-related tasks posed, in an effort to amplify the likelihood that students will exhibit *empirical* and *analytical schemes* while doing proofs.

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