CONTEXT COMPLEXITY AND ARGUMENTATION

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This paper elaborates on the hypothesis (taken from psycholinguistics research) that the development of argumentative skills depends on contextual factors. With reference to Mathematics Education, the potential inherent in context complexity is investigated, distinguishing between different kinds of complexity. Some educational implications concerning the teacher's didactical choices are sketched.

The importance of argumentation [1] in Mathematics Education has been increasingly recognised in the last decade. In spite of the development of research concerning the 'why' of argumentation, a relatively small number of papers dealt with the 'how', in particular with the conditions suitable for the development of argumentative skills; only few dealt with the mathematical and extra-mathematical content of the problem situations as a relevant factor. However, there are good reasons to think about the relevance of these 'context' factors (for a discussion about the meaning of 'context' see the next Section). In psycholinguistics the hypothesis of context-dependence of argumentative skills was an object of widespread investigation during the 70s and the 80s (for surveys, see French, 1985; Sell, 1991). The aim of this paper is to investigate the potential of context complexity in the development of students' argumentative skills related to mathematics at primary school level. I will also draw attention to some conditions (as regards the teacher's choices) for the potential of context complexity to be exploited. The orientation of this paper is theoretical. However two teaching experiments, performed in the past with different purposes, will be shortly presented; they will provide the reader with some examples to ground the analysis.

THEORETICAL FRAMEWORK

The general hypothesis underlying the research reported in this paper is that the quality of students' argumentation strongly depends on the nature of the objects and situations which argumentation refers to, provided that suitable didactical choices (concerning the tasks and the educational environment) are performed by the teacher. This hypothesis draws on the context-dependence of peculiar forms of argumentation. Considering the contributions brought by *"several major developmental theorists"* (Brown, Donaldson, Gelman, and others) to research concerning contextual factors in cognitive development, L. French wrote: *"They are in basic agreement about the importance of contextual factors in both the acquisition and display of cognitive abilities. Their essential premise is that cognitive competence initially arises within, is embedded within, and is practiced within particular contexts"* (French, 1985, p. 321). Referring to specific psycholinguistic research concerning the acquisition and display of cognitive competencies related to the comprehension and use of relational terms ('because', 'if', 'but', 'or', etc.) she stressed the importance of *"the content of sentences in which relational terms are embedded"* (ibidem, p. 322).

As regards the meaning of the word 'context' in this paper, we can observe that, in Mathematics Education as well as in common language, the word 'context' may take different meanings (for a survey see Wedege, 1999, pp 206-207). It can refer to constraints, signs, situations that must be considered by the student in accomplishing a given task ('reference context'); to the formulation of the task (context as 'text'- Freudenthal, 1991); and to the environment – in particular, the educational environment - where the activity takes place (the 'situation context' - see Wedege, 1999). In our hypothesis all the different meanings of the word 'context' intervene, and 'reference context'-dependence is not seen as isolated from 'situation context'-dependence. In the next Section we will consider some aspects of the 'reference context' (simply named **'context'**), while some educational implications concerning the 'text' and the 'situation context' factors will be sketched in the last Section.

As concerns the argumentative skills considered in this paper, we will refer to the mastery of relational terms as well as more complex skills that intervene when contradictions are detected and made explicit, when coherence is questioned, etc. Tasks and argumentative skills will be considered in the perspective of mathematics education at primary school level, with an eye to more advanced mathematics.

In this paper, the potential of context complexity will be considered as regards both the number of elements to be co-ordinated in a problem situation (or in a set of tasks dealing with the same subject) and the quality of the links to be established (from simple juxtaposition to comparison, up to a comprehensive and coherent systematisation) (cf. Sell, 1991). This paper will elaborate on the specific educational hypothesis that primary school students' argumentative skills can be enhanced by exploiting context complexity in two ways: for some students, as an opportunity of exercising specific argumentative skills; for other students, as an opportunity of approaching these skills through the interaction with more competent peers and/or the direct mediation of the teacher, who suggests the appropriate linguistic forms to express complexity (cf 'Zone of Proximal Development': Vygotsky, 1978, Chapt. VI).

EXAMPLES

Concrete examples illustrating the ideas developed in Section 4 will be taken from two teaching experiments performed by the Genoa Group Project in the years 1997 and 1999. The first experiment was carried out in a grade-II class and concerned the task of measuring the height of wheat plants growing in a pot with a ruler having a one-centimetre distance between the edge and the beginning of the scale: *"How can we measure the height of the plants in our pot with our ruler, in order to account for their growth over time?"*. From the educational point of view, the task was aimed at constructing some operational invariants (Vergnaud, 1990) of the concept of measure, e.g. the translation invariance and the additivity of lengths. From the research point of view, the aim of the teaching experiment was to study the argumentative roots of the construction and refinement of those operational invariants up to an explicit and conscious mastery of them. The research was reported in Douek and Scali (2000).

The second experiment concerned the 'Stefano's problem' in a IVth-grade class:

> *"At the beginning of classwork on sun shadows, Stefano (a VI grade student) thinks that shadows are longer when the sun is higher and stronger. Other students think the contrary. In order to explain his hypothesis, Stefano produces the following drawing:*

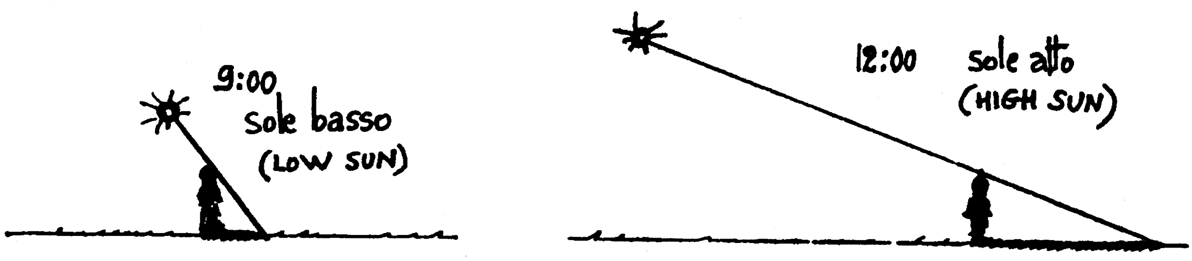

Fig. 1

and writes: "As we can see in the drawing, the sun makes a longer shadow when it is higher, that is at noon, when it is also stronger"

We know very well that shadows are longer when the sun is lower (early in the morning and late in the afternoon). So, in Stefano's reasoning there is something that does not work. What is wrong with Stefano's reasoning, and particularly with his drawing? Try to explain yourself clearly, so that Stefano can understand."

From the educational point of view, the aim of the task was to refine the model of the Sun shadows phenomenon (the 'shadow schema') already used by students in many activities. The task put into question the meaning of the word 'high' when referred to the 'height' of the Sun in the sky. A link with the concept of angle was implicit in the task. From the research point of view, the aim was to study the process of conceptualisation and the role of argumentation in it (see Douek, 1998 and 1999a).

In both experiments individual tasks were alternated with classroom discussions about individual productions, managed by the teacher. The experiments were performed in an educational context where verbal reporting of thinking processes (with no worry about their correctness) was an important element of the didactic contract.

SOME KINDS OF CONTEXT COMPLEXITY, AND ARGUMENTATION

Different kinds of context complexity will be considered in relation to their specific potential in the development of argumentative skills. We will move from complexity depending on the characteristics of the content of the problem situation, to complexity inherent in the tools needed to tackle the problem.

Time and Space Complexity

In mathematical modelling activities, objects and situations to be dealt with can belong to different space scales (micro-space, meso-space, macro-space: see Berthelot&Salin, 1992). They can also be involved with different time scales. We can observe that the passage from one space (or time) scale to the others entails relevant changes in cognitive processes. Complexity depends on the need for a comprehensive consideration of phenomena, which are to be analysed under different time and space scopes. The specific forms of argumentation which intervene in managing time and space complexity concern comparing arguments related to different space or time environments, or checking the validity of a statement according to different space or time scales, (etc.), particularly under the form of hypothetical reasoning.

In the specific case of 'Stefano's problem', space complexity intervenes when students must create links between the drawings (made in the micro-space of the sheet of paper), the observed phenomenon (happening in the meso-space of the courtyard experiment) and the phenomenon occurring in the macro-space of the sun system. During a classroom discussion concerning the problem of the 'command variables' of the geometric model of shadows, the teacher asked what would happen to his own shadow if he moved one step forward. He meant to put into question the relation between the virtual observation of his shadow if he moved forward and the schema drawn on the blackboard. A 'good' student, Federico, transferred to the macro-space what he could imagine in coherence with the schema on the blackboard, where the sun was drawn near the obstacle that cast the shadow. He said: *"If you move, then your shadow becomes longer, because the Sun stays there and you go farther"*. The conception of the relative positions of sun, earth and obstacle in the macro space that had been developed during the experimental work in the meso-space of the courtyard, was replaced by the direct observation of the micro-space schema. As a consequence, it was then necessary to rebuild coherence between the previous conception and the schema. The evident contradiction of Federico's statement with what many students knew (the invariance of the length of the shadows of a moving object) provoked high quality argumentation in the classroom. Some excerpts are reported below:

Simone: *"I agree that the shadow will become longer; if you move in that drawing, the Sun does not move and you can see that the shadow becomes longer and longer"*

Andrea: *"I do not agree with Simone and Federico, I can give you an example: if you are in a car and move... for instance, from Turin to... to Pinerolo... you see the sun...always with the same height in the sky... and the shadow does not change. The position of the sun changes only if a long time elapses! So there is something that does not work in this drawing."*

Mariella: *"In the real world, near to the Earth, sun rays are parallel, while in this drawing... using this drawing ... they are convergent".*

Anna: *"Yes, the sun is not there... it is very, very far"*

Coherence was gradually reconstructed through argumentation by relying on different sources of arguments, as described in details in the next Section.

Complexity of Sources of Arguments

Arguments may originate from the classroom history, from empirical or theoretical evidence, from shared principles, etc. Complexity depends on the need for integrating different sources and making them coherent, when for example one source is not sufficient to get a satisfactory solution, or when some contradictions emerge. This need for integration and coherence requires specific forms of argumentation; the *"A, while B"*, *"A, but B"* and *"if A then B so C"* clauses intervene as necessary tools.

Let us go back to Federico's confusion. A contradiction appeared between his statement, based on the 'shadow schema' drawn on the blackboard, and other references shared by his schoolmates. First, it clashed with another geometrical model previously elaborated in order to solve the specific problem of the effect of changing the position of the obstacle in the courtyard on the cast shadow. In that model the sun was not drawn; a series of parallel rays on the sheet of paper showed that wherever the obstacle moved, the rays cast shadows with equal lengths (see Mariella's utterance). Second, it clashed with the content of a classroom discussion about the 'shadow schema' in which it was agreed (by moving the sun farther and farther) that, since the sun is very far from the Earth, therefore sun rays can be considered parallel, and not divergent from their source, when they reach the Earth (see Mariella's and Anna's utterance).

We can remark that the complexity of the situation was successfully faced by the multiplicity of voices (it would have been difficult to tackle it individually!), and this was necessary to bring the different pieces of knowledge together coherently.

Static-Dynamic Complexity

In applied mathematical problem solving, dynamic situations must be represented in a static way in order to elaborate the relations that connect relevant variables. For instance, in the case of Stefano's problem the dynamic evolution of the sun shadow phenomenon is usually represented (in the students' individual solving processes) through two or more drawings concerning specific states of the phenomenon itself. Complexity derives from the need for relating static representations to one another and to the evolution of the phenomenon in a coherent way. The argumentative skills involved concern the discussion of the coherence between representations and reality.

In particular, in the case of 'Stefano's problem', one student, Ambra, drew a pair of

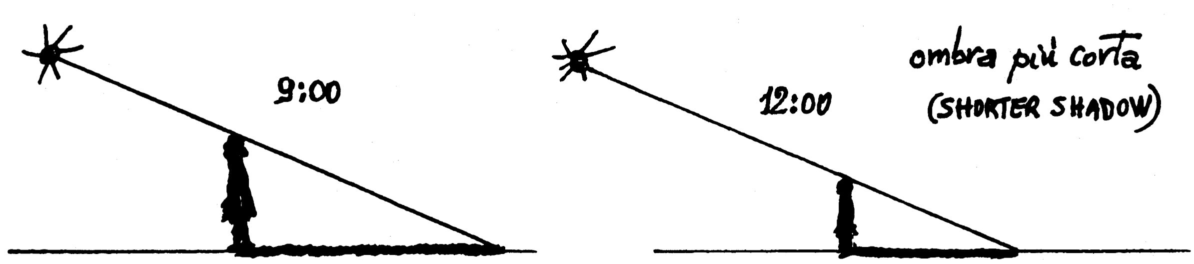

Fig. 2

schemas in which the position of the sun and the inclination of the sunrays remained unchanged; instead she changed the height of the obstacle casting the shadow in order to obtain a *shorter shadow at noon* (this was a shared knowledge in the classroom). Ambra did not relate consciously the different hours of the day to the different positions of the sun in the sky. At that moment she lacked arguments to choose another variable (the slope of the sunrays) and produce schemas coherent with her past experiences. The slope of the sunrays appeared as crucial during the following discussion, and various *connected* arguments were considered. In particular, the students recalled a past experience; with the help of the pictures taken in that occasion they represented the slope as the inclination of the arm pointing at the sun *"moving in the sky"* in different imagined positions. So a link with the static representation of the 'sun shadow schema' was established. Some excerpts are reported below:

"Ambra's drawings do no work together, because the heights of the child are different, while in the real world the height does not change"

" It is the slope of our arms when we imagine ourselves pointing at the sun moving in the sky that changes"

"It is the slope of our arms in the pictures that changes, and so it is the slope of the rays in the sun shadow triangle that must change ".

Complexity Inherent in the Change of Frames

A problem situation can be considered within different frames (for 'frames' in Mathematics Education, see Arzarello et al, 1995). For instance, some difficult Euclidean geometry problems can easily be solved within the frame of analytic geometry. Complexity depends on the need for comparing different frames and moving through argumentation from one to the other.

In the case of the 'Wheat plants measuring problem' this kind of complexity intervened during the individual solution phase, in which the students interacted with the teacher. The first thing most students wanted to do was pushing the ruler down into the pot, or breaking the bit before the zero (a concrete, physical solution). Then the teacher argued against such solutions (because they would damage either the roots or the ruler), so the students gradually switched to *imagining* to push the ruler (or to break it). The teacher drew their attention to the number they could read when the ruler was just beside the plant and the one they would read if the ruler was pushed down (or broken). The students had to consider both what they read and what they would read. Many of them developed solutions: either they imagined the numbers were all sliding, one after the other ('translation solution'), or they imagined to stick the little broken piece at the top of the plant ('additive solution'). They arrived to a solution within a material physical frame, consisting of virtual actions on real objects. When they came to writing their solutions or giving examples, in order to explain their solutions, they often moved to the numerical frame, and said *"It is the following number"* or *"I add one to the number I can read".* The physical solution was a necessary step to gradually get to a 'translation' or an 'additive' solution within the numerical frame. Afterwards, the solutions were shared in the classroom (see next Section) and a discussion followed. At the end, the students developed complex argumentation in order to account for the validity of each solution and compare the two solutions:

"In our case Rita's reasoning works, because 1 on the scale goes to 0 on the ground, and then 2 goes to 1, 3 goes to 2, and finally I can read the exact measure. But this shows that it works well because the initial piece of the ruler is 1 cm long. Alessia's reasoning always works because it is as if I broke it and add the initial piece, and I perform the same thing with numbers. And I can always do the same and add the number accounting for the measure of the initial piece, whatever it is".

Complexity of External Representations

A plurality of systems of signs (verbal, iconic or geometric, symbolic) may intervene in many mathematical activities (such as modelling). Complexity depends on the necessity of co-ordinating such signs and dealing with them in a functional way (cf Duval, 1995):

for instance, language as a command and reflective tool, geometric signs as models (as in Stefano' problem), algebraic symbolism as computational devices, etc. In collective discussions this kind of complexity involves different games of interpretation and related argumentative skills, especially concerning the control of solution strategies.

In the 'Wheat plant measuring' activity, after the individual problem solving (in interaction with the teacher) and presentation of the 'additive' and 'translation' solutions, the students were provided with two drawings (to stick in their copy books) representing a wheat plant in its pot, and two paper rulers. They were first asked to express clearly (in written words) the 'translation' solution, and they all did it within the physical frame. Then they had to represent the translation of the numbers on the ruler with a sequence of arrows; and then to write the calculations that represented symbolically in the numerical frame what they got in the physical frame. They repeated this kind of representation for the 'additive solution'. This activity allowed intensive argumentation in interaction with the teacher, because the teacher systematically asked to justify the arrows and the calculations with reference to the imagined virtual solutions within the physical frame. These solutions played the role of justifying the choice of the calculations that solved the problem. Here is an example of an individual text (after the interaction with the teacher):

"In words I can say that I move the ruler, so each number slides, it is as if each number vanished and became the following one: 0 becomes 1, and 1 becomes 2, and so on. If I have to show it I must draw arrows from 0 to 1, from 1 to 2. But to get the measure only the last step is important: I read 22 on the ruler, then 22+1 is the measure".

The same complexity in dealing with various representations can be found in the discussion about Ambra's drawing in the 'Stefano's problem'.

SOME REMARKS ABOUT THE ROLE OF THE TEACHER

The exploitation of the potential inherent in the contextual elements described in the previous section relies on appropriate didactical choices made by the teacher.

First of all, we must consider the importance of the choice and formulation of tasks, and the choice of the right moment to perform them in the classroom, in order to avoid that complexity is reduced. Concerning the choice of tasks: A problem like the 'Stefano's problem' involves different kinds of complexity, with an increased potential for argumentation. Concerning the text: In some experiments performed within the Genoa Group Project we have seen that the 'Wheat plants measuring' activity looses most of its potential if the following text is given to the students: *"Keeping into account that plants cannot be extracted from the ground, that the ruler cannot be put into the ground and that the ruler has a one-centimeter distance between the edge and the scale, establish how to measure the height of wheat plants in the pot"*. We saw that the percentage of good solutions decreased and the quality of the classroom discussion was lower. Probably, the formulation of the task guided the solution process and prevented students from detecting and managing the constraints of the problem situation. Moreover it is obvious that the argumentative potential of the problem situation cannot be exploited if the task is proposed too late.

Second, we must consider the management of classroom situations. As regards Stefano's problem, another teacher tried to exploit the same problem in her class. As a reaction to students' initial difficulties, she decided (before the classroom discussion about students' individual productions) to suggest the use of the concept of angle, saying that *"the mistake of Stefano was to consider the height of the sun as a distance from the ground, not as the angle formed by the sunray with the ground"*. The consequence was that no real debate was developed during the following discussion and the students simply wrote (after the discussion) a more or less precise paraphrase of what the teacher had said.

NOTES

[1] In this paper, the word **argumentation** indicates two things. First, it denotes the individual or collective process that produces a logically connected, but not necessarily deductive, discourse about a given subject. This agrees with the first definition for 'Argumentation' provided by the Webster Dictionary: *" 1. The act of forming reasons, making induction, drawing conclusions, and applying them to the case under discussion"*. Second, it points at the text produced through that process, in accordance with the meaning the Webster dictionary presents: *" 3. Writing or speaking that argues"*. The linguistic context will allow the reader to select the appropriate meaning, whenever this word is used. In this paper, the word '**argument**' is used with the meaning of *"A reason or reasons offered for or against a proposition, opinion or measure"* (Webster) . So, an 'argumentation' consists of one or more logically connected 'arguments'.

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