THE EVOLUTION OF A STUDENT TEACHER'S PEDAGOGICAL VIEWS ABOUT TEACHING MATHEMATICS PROOF

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Abstract. *This paper analyzed the change of a student teacher's pedagogical views about teaching mathematics proof during four years in a teacher education program. The changes identified in this study have shown a shift of her pedagogical views from convincing-formal to discursive explanatory. The changes of her related views about proof from formal to explanatory have been observed, too. Her change can be interpreted by Cooney's model of student teachers' belief change. A reason underlying her change is due to her participating in an action research in which she was becoming a reflective practitioner.*

INTRODUCTION

Issues such as the role and function of proof (Villiers, 1987), the view about proof (Lerman et al., 1993), teaching approaches of proof (Hoyles & Healy, 1999; Sekiguchi, 2000) and learning theory of proof (Duval, in press) have been concerned and studied. Villiers (1987) found that about 60% of pre-service mathematics teachers realized the function of proof only in terms of verification/justification/conviction, and were not able to distinguish any other functions of proof. Studies of student teacher's views about teaching proof indicate that pre-service teachers' interpretations of mathematics proof differed from what the mathematics community would consider as mathematically acceptable (Knuth & Elliott, 1997). Their expectation of proving by their students was limited and they have a different notion of what is an acceptable mathematical proof outside of the formal setting of their university mathematics classes. Wittmann (1992; 1996) distinguished demonstration from proof, the former often having a very concrete nature, and argued for introducing demonstrative reasoning as soon as the first grade. He suggested that operative proofs should be included in primary mathematics.

Those studies could help us on understanding different pedagogical views about teaching mathematics proof. But how did the student teachers' pedagogical views about teaching proof evolve during the teacher education program is still unclear. The relationship between the view about proof and the pedagogical view about teaching proof and what is the mechanism to promote the evolution of student teachers' pedagogical views about teaching proof needed to be clarified. Those problems are what we are studied in this paper.

METHODOLOGY

This study was part of the research-based pre-service Mathematics Teacher Education Project, an inquiry into the development of pedagogical power of pre-service secondary mathematics teachers. One of two principles about mathematics teaching posed by the educator in the teacher education program is teaching for making sense of mathematics (Lin, 1999). The courses, mathematics learning, mathematics teaching and evaluation, mathematics content and method, and teaching practice, were included in this program.

The program could be divided into three stages: entering the program during the sophomore year (stage 1), practicing teaching during the junior year and senior year (stage 2) and being a beginning teacher for one year (stage 3). In the course of mathematics learning (stage 1), student teachers were required to investigate students' understanding of some mathematics concepts and discuss strategies to help students' learning. In the courses of mathematics teaching and evaluation, mathematics content and method, and teaching practice (stage2), the main activities were creating student teachers' experiences on different teaching approaches such as guided discovery, group discussion, and investigative teaching, via activities of analyzing mathematics materials and designing learning activities, and provided opportunities to reflect on expert teachers and peers teaching. The aim through the program is to develop student teachers' pedagogical powers about teaching mathematics. At stage 3, the selected student teacher for the study taught mathematics in a secondary school and conducted an action research with us. There was a mentor in the school who should give suggestion to her but he did not do so.

The methods used in this study include questionnaire survey, classroom observations, and semi-structure interviews. A questionnaire designed to elicit student teacher's views about proof and pedagogical views about teaching mathematics proof was administered to the student teacher Echor at stage 1, 2 and 3 (1998.03-1999.05-2000.08). The questionnaire had two tasks. The first task was modified from Lerman et al., 's (1993) study: "the sum of the interior angles of a triangle is equal to **180° ".** The second task adopted Hanna's (1983) three ways to prove $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$ and added four typical solutions collected from Taiwanese students. The student teacher, Echor, was asked to choose which way she preferred, give comment and to rate the quality of each solving method with scale of 1 to 5 for every item and choose which ways she preferred to use in her classroom.

Classroom observations and semi-structure interviews were conducted (stage 1--stage 3) and verbatim transcribed. In addition to those, we have collected her reflection notes and learning profiles during her pre-service education. All data were confirmed by Echor later and used as a means of gleaning additional evidences of the questionnaire results.

Hanna's (1990) and Hersh's (1993) taxonomy of proof: formal proof and explanatory proof was adopted as an initial scheme to characterize student teachers' views about proof. After the data of classroom teaching were analyzed, it was found that formal proof could be divided into two categories, formal view and convincing-formal view, and explanatory view could be divided into two categories, instructional explanatory and discursive explanatory, according to the teaching approach, that is teacher-centered or is student-centered respectively. Thus four pedagogical views about teaching proof are characterized as follows.

1. formal view (for convincing): teacher demonstrates well-organized deductive statement.
2. convincing- formal view (for convincing): teacher convinces students the truth by manipulation and special case, and follow with formal proof.
3. instructional explanatory view (for understanding): teacher demonstrates explanation.
4. discursive explanatory view (for understanding): the explanation is resulted from students' discourse.

Those four views distinguish how teacher understand the functions of proof and what are the approaches of teacher's teaching.

ECHOR'S PEDAGOGICAL VIEWS ABOUT TEACHING PROOF

This section reports the student teacher's responds on which ways she would use in her teaching with reasons. The following table categorizes the ways Echor chose on each task at different stages.

Tasks	**Stage 1**	**Stage 2**	**Stage 3**
Task 1: the sum of interior angles	1-2: Measuring 1-5: Arnauld's 1-3: Euclid's	1-1: Paper & Scissors 1-4:Logo-style	1-1: Paper & Scissors/ 1-2: Measuring 1-4: Logo-style
Task2: 1+2+3+…+n $=\frac{n(n+1)}{2}$	2-7: Mathematical induction 2-2: Induction 2-5:Triangular numbers 2-6: Staircase-shaped area	2-1:**modified** Gauss's	2-1: Gauss's 2-3:Arithmetic formula 2-5 and 2-6 are used conditionally

Fig.1 Categorizing the ways chosen at different stages

At stage 1

Refer to Fig.1, Echor chose multiple methods to prove the tasks. Her comment on measuring method was that it is inaccurate. She preferred formal proofs and convinced students the statement is true by manipulation. She said:

" *I must help students to make sense (1-2) first and then show them one* *or two formal proofs (1-5, 1-3, 2-7). After these procedures, I will give more explanations (2-5, 2-6) to them finally."*

Hence, her pedagogical view is toward to convincing- formal.

The above data was compared with her interview data at the beginning of this stage about teaching mathematics (Chin& Lin, 1998) .It was found that she emphasized students' needs should be considered first, and teacher's instruction ought be accommodated to meet halfway with their students. She said that students' learning should be the priority but she described her teaching design with teacher-centered approach. She chose formal proofs to teach the two tasks, and added teacher-centered empirical verification for making sense before proving. Thus, we classified her pedagogical view about teaching proof as **convincing- formal view** at stage 1.

At stage 2

Echor chose Paper & Scissors way as empirical verification for making sense. Logo-style was in the unique Taiwanese textbook and was popularly used in Taiwan mathematics classrooms. Gauss's method was the genetic method to prove task 2. The word "modified" means adding several numerical examples for testing. Her choice changed from multiple methods to a single one. All the methods of proving she chose were explanatory. Therefore, her pedagogical view was explanatory.

Data cited from our analysis of her teaching practice at stage 2 indicated that she taught proofs by lecturing without providing opportunities for her students to be engaged. She emphasized on making sense of mathematics for her students, and carried out her idea with teacher-centered way. For example, when she taught the theory of the sum of exterior angles in a triangle, she said everything by herself and just asked: "*Do you understand?*". She tried to implement the ideas she have learned from the teacher education program to her classroom, but her ideas did not come true in her teaching practices. Her pedagogical view at stage 2 was classified as **instructional explanatory**.

At stage 3

Refer to Fig.1, Gauss's method was used to explain Arithmetic formula. Her pedagogical view was explanatory. By contrasting her teaching at stage 3, we found that there were many activities for conjecturing, discussing, reporting, and interacting with the students. For example, when she taught polygonal pyramid, she said: "*how do you feel about this polygonal pyramid*?" "*Please make a conjecture: how many vertex, edges, and faces does a polygonal pyramid with a regular polygon base have*?" When her students make an error in constructing an identical line, she asked: "*why do you know they are identical?*" and "*when are they identical?*" Echor posed questions to guide students thinking and encouraged students to talk about their ideas. Her pedagogical view at stage 3 was classified as **discursive explanatory**.

THE CHANGE OF ECHOR'S PEDAGOGICAL VIEWS

From stage 1 to stage 2, the changes included that she changed her comment on induction, pointed out the function of the connection between algebra and geometry concretely, and was able to adopt strategies to fit individual difference. She agreed that using figural representation to explain identity has its positive meaning—it was intuitive and concrete. But she would choose numerical examples in her classroom teaching rather than the number-shape transformation under the consideration of its complexity and the rigorous of proof. Echor recognized mathematical induction was not easy for students to learn because of it was with little explanatory value. Mathematically, she was aware of the rigorous of mathematical induction, but she would not use it in her teaching. Her criterion was the simplest the best. The interview data about her change was as follows:

> "*I think my big change is that I try to do my best to help students getting a lot of knowledge before. So I use many ways to prove one thing. But I think one method is enough now. For beginning learner, one simple method is good enough.* "

From stage 2 to stage 3, her change was unclear if we just analyzed data in Fig.1. All the ways she chose were explanatory proofs like she did at stage 2. We analyzed her classroom teaching and found that she have realized the function of proof for explanation, testing, and justification (Yackel, 1998; 2001). She changed her view from instructional explanatory into discursive explanatory and adopted different criteria of rigorous for professional mathematics and classroom context respectively.

Two reasons are underlying her change. The feedback from students during her teaching practice in school for about four weeks at stage 2 have stimulated her to reflect that rigorous is not the first thing for student to learn at secondary level. She said:

> "*The experiences of teaching had an important influence on my view about teaching proof. When I responded on the questionnaire first time, I just considered which one had the most beautiful structure from my point of view. …… After I have taught, I knew that if I teach following the beautiful ways in the questionnaire, only very few students could understand.*"

Another reason is her reflection on the processes of the action research. She said:

> "*After this experience, I knew that the platform should not be occupied by me. Students need platform, too. Teaching could be implemented in different way and students could learn actively. I think…I should wear their shoes to teach mathematics from their views….*"

Her reflection on teaching in the action research at stage 3 helped her to be aware that student-teacher interaction may be a good approach for teaching and helped her to become a reflective practitioner.

ECHOR'S VIEWS ABOUT PROOF AND VIEWS ABOUT TEACHING PROOF

We analyzed Echor's responses on the questionnaire about proof as Fig.2 shown. The change of her views about mathematics proof at different stage was from formal view, to formal view, and then to instructional explanatory view.

Questionnaire Items	**Stage 1**	**Stage 2**	**Stage 3**
Problem 1-- the sum of interior angles			
1-1: Paper & Scissors	2	2	3
1-2: Measuring	2	1	3
1-3: Euclid's	5	5	4
1-4:Logo-style	4	4	5
1-5: Arnauld's	5	5	4
Problem2 -- $1+2+3+\ldots+n=\frac{n(n+1)}{2}$			
2-1: Gauss's	5	5	5
2-2: Induction	3	3	1
2-3:Arithmetic formula	4	5	5
2-4:Trapezoid formula	3	3	1
2-5:Triangular numbers	4	3	5
2-6: Staircase-shaped area	4	4	5
2-7:mathematical induction	5	5	5

Fig.2-- Different ratings given for different ways of proving the tasks at different stages

Refer to Fig.2, Echor's views about proof kept consistence from stage 1 to stage 2. She rated high scales to formal methods. Observing the flow of the scales from stage 1 to stage 3, it could be seen that the weight of explanatory proof was raised. The weight of formal proof became less and the weight of explanatory proof became higher. At the end of stage 3, Echor emphasized the function of explanation in a proof, understood the cognitive function of informal ways, considered the cognitive needs of the students and then gave up the formal way she preferred. We conjecture the change of her views about proof was resulted by the change of her pedagogical views about teaching proof.

DISCUSSION

Cooney et al., (1996) posit a scheme to conceptualize the professional development of pre-service secondary mathematics teacher. They regarded teacher's development as a means for categorizing teachers, i.e. four positions: isolationist, naive idealist, naive connectionist, and reflective connectionist. Those four positions reflect the extent to which teachers resisted or accommodated new teaching methods into their teaching schemes and exhibit a reflective orientation toward mathematics and teaching. Those four positions mentioned above could be used to describe Echor's development of pedagogical power in this study. At stage 1, she accepted the authority suggestion and wanted to use many ways to enhance students' understanding because she believed that every way of proof had its explanatory value and could contribute to students' learning. This showed she was a **naive idealist.** At stage 2, she just chose a simplest way for teaching proof. This decision was made pedagogically. She was aware that most of students in her class were confused with multiple methods for proving one statement. She can connect mathematics with her pedagogical mathematics. She also could identify the tensions resulted from the difference between her views of proof and about teaching proof. But she failed to see the significance of the connections and made no attempt to resolve the identified tensions. Thus she was a **naive connectionist**. At stage 3, she was becoming a **reflective connectionist** because she accommodated pedagogical ideas as her belief system that was restructured. The pedagogical view that was accommodated showed that she changed her view into explanatory, interactive, and discursive after her belief system was reconstructed. The change was not only the shift of her belief system about teaching mathematics proof but also the change of her belief about mathematics proof. The development of Echor's pedagogical power could be described as starting from a naive idealist, transferring to a naive connectionist, and finally growing to a reflective connectionist.

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