ANALYSIS OF THE TEACHER'S ARGUMENTS USED IN THE DIDACTICAL MANAGEMENT OF A PROBLEM SOLVING SITUATION

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ABSTRACT: *In this paper, we analyze an investigative situation proposed to a class of 5th graders in a primary school. The situation is based on the following task: In a sale with group rates on a sliding scale, the students must find the lowest possible purchase price for a given number of tickets. The aim of this paper is to show that one of the intrinsic features of the situation restricted the teacher's possibilities of making didactical use of the students' forms of reasoning processes during whole class presentation and discussion of the reports.*

1. INTRODUCTION

The study presented in this paper is a part[2] of an article on the role of the different forms of reasoning in the didactical relation, in mathematics, at the primary school level.

We start by explaining what we mean by "reasoning" (section 2). The term is widely used by teachers of all subjects and by researchers, with a variety of meanings.. Therefore, we had to directly define the object and the methodology of our study before classifying the different forms of reasoning we were concerned with.

In section 3, we will present the problem situation observed and in section 4, we will identify several forms of reasoning which appeared in class during students' investigation [in small groups] and subsequent whole class presentations and discussions.

In section 5, we will address the following questions:
Did the proposed problem situation favor students' production of forms of reasoning?
Which didactical decisions of the teacher strongly determine the presence, the meaning and the actual possibilities of processing and using students' forms of reasoning?

2. REASONING IN THE CLASSROOM

2.1. Actual forms of reasoning

We define a *reasoning* as a relation *R* between two elements A and B such that,
- A denotes a condition or an observed fact, which could be contingent upon particular circumstances;

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[2] It is based on a set of conceptions and results which have been presented in more detail in Guy Brousseau and Patrick Gibel:"Didactical handling of students' reasoning processes in problem solving situations", Educationnal Studies in Mathematics (2005) 59; 13-58.

- B is a consequence, a decision or a predicted fact;
- *R* is a relation, a rule, or, generally, something considered as known and accepted.

The relation *R* leads the acting subject (the reasoning "agent"), in the case of condition A being satisfied or fact A taking place, to make the decision B, to predict B or to state that B is true.

An *actual* reasoning contains, moreover,
- an agent E (student or teacher) who uses the relation *R*;
- a project, determined by a situation S, which requires the use of this relation.

We can say that to carry out a project determined by a situation S the subject uses the relation *R* which allows him to infer B from A. This project can be acknowledged and made explicit by the agent, or it can be attributed to him by the observer on the basis of some evidence.

2.2 First classification of forms of reasoning according to their function and type of situation

As implied in the previous section, reasoning is characterized by the role it plays in a situation, i.e. by its function in this situation. This function may be to decide about something, to inform, to convince, or to explain. The function of reasoning varies according to the type of situation in which it takes place; on whether it is a situation of action, formulation, validation or other (Brousseau, 1997: 8-18).

3. THE OBSERVED LESSON

3.1 The components of the situation

The lesson took place in a 5^{th} grade mathematics class.

3.1.1 The problem and the objective situation

The teacher starts by handing out the following problem:

> A one-day ski trip to the resort of Gourette is being organized next Saturday for students from the Oloron area. For this exceptional event, the local city council has decided to pay for the ski passes for the day. The resort of Gourette offers the following group rates:
> 216 passes: 1275F
> 36 passes: 325F
> 6 passes: 85F
> 979 children have signed up for the trip but when the morning of departure arrives 12 children do not turn up because they are sick, of course. The council accountant says to himself "Too bad for these kids, but never mind, it'll work out less expensive for us this way".
> What do you think?

The "objective situation" is the situation presented in the problem; the student is expected to deal with it without questioning the status of reality or not of what is thus presented to him as "objective".

3.1.2 The planned phases of the lesson

The development of the lesson, chosen by the teacher, follows a plan that has become quite common in France:
- the research activity is presented by the teacher (phase 1);
- students read the problem (phase 2);
- the teacher provides additional information, if necessary; for example - explains the terms used in the formulation of the problem (phase 3),
- students work on the problem individually for about 10 minutes (phase 4),
- students are divided into small groups (phase 5),
- students work in small groups, and prepare a written report; this phase (phase 6), lasts about 25 minutes;
- whole class presentation and discussion of the reports, with each group going to the board in turn to present their results (phase 7).

3.2 How the lesson developed

3.2.1 The research activity and the written traces of it In the observed lesson, the research activity was based on the research and formulation of the question, which completely determines the problem (in the classical sense of the term). But the students were not able to perceive what is at stake (mathematically) in the problem situation and it is the teacher himself who formulated the question: "When, do you think, is the ski trip more expensive: when there are 979 students or when there are 967 students?"

3.2.2 The phase of whole class presentation and comparison of students' solutions

Our theoretical, *a priori*, analysis of the problem situation led us to expect a failure of the teacher's plan: The management of the didactical phase of the lesson (phase 7) appeared all the more delicate that the reduction of the complexity was essentially in the hands of the teacher; it depended on his choices, his decisions and his "opportune" interventions.

But upon viewing the video recording of the lesson (which we haven't seen before the theoretical analysis), we had to admit that the teacher managed to conduct his class without being challenged with any major difficulties.

4. THE OBSERVED FORMS OF REASONING, THEIR FUNCTION AND USE

4.1 Forms of reasoning in students' written productions

The analysis of the different forms of reasoning which appear in the students' solutions shows that what is really at stake in the problem situation, namely the problem of minimizing the expense, has not been grasped by the majority of students.

In this lesson, it is clear that the devolution of the situation did not work; the students were not able to take charge of the proposed situation. Indeed, in the phase of whole class discussion and comparison of solutions, it appears that:

- The students do not possess the necessary knowledge to conceive of the basic strategies.
- The students cannot obtain, as feedback to their actions, the information necessary for the solution of the problem.
- There is not enough time for the students to produce a solution, because of the complexity of the problem.
- The students have no means to judge, by themselves, the validity of their solutions.

4.2 Analysis of an episode of interactions during the whole class discussion phase

For this paper, we have chosen to present an analysis, in terms of the theory of didactical situations in mathematics, of an excerpt from the transcript of phase 7, i.e. the whole class discussion and comparison of students' solutions phase.

The episode focuses on interactions related to one student's work. This student, Julien has chosen to work alone. His written work is presented in Figure 1.

Our analysis of this episode is presented in Table 1. The first column of the table contains the code of the intervention, where the first number (4) indicates that Julien's "small group" (composed of him alone) was the fourth to present its results. For some interventions, the timing is shown (since the beginning of phase 7). The second column contains the transcript, and the third some comments on the intervention. In the fourth column we analyze the nature and the function of the intervention with regard to the locutor's intended project. The fifth column aims at articulating the function of the intervention.

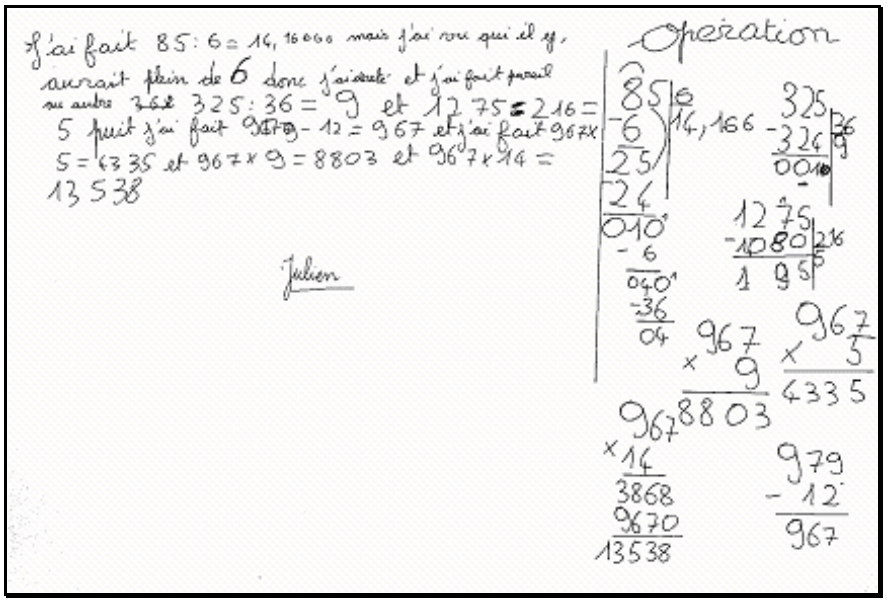

Figure 1

TABLE 1. Transcript and analysis of some interactions

N° Min.	**Transcript**	**Comments**	**Analysis**	**Nature and function of the intervention**
4.1 12'35	Julien: Okay, I started by doing… (1) I divided 6 into 85 (2) and I got 14,166;	Julien comes to present his work. He describes his calculation, without defining or naming the		(1)Direct description of an action (calculation) (2)Formulation of a result (3) Indirect reference to an action: by analogy (4) Organization of the calculation

	I took the 14 and then I saw… (3) I did the same with 325, in short, I did the same, (4) I did the same with all three operations	variable that he calculates.		Strategic or organizational reasoning, local and expressed orally.
4.2	Teacher: (1) the three proposals, (2) the group passes	The teacher reformulates a part of the student's statement to introduce a vocabulary.	The teacher wants to establish a link between the performed calculations and the objective situation.	(1) Correction of the terminology (2) Suggestion of a terminology and giving a name to a result.
4.3	Julien: (1) 325 divided by 36 and 1275 divided by 216 (2) and then I did…	Julien continues to describe his calculations		(1) Direct description of an action (2) Organization of calculation. Strategic or organizational reasoning, local and expressed orally.
4.4	Teacher: (1) [Your] first conclusion after these calculations? [to the whole class] Have you heard the operations he had done? (2) What is the price of a pass, relative to each of the three proposed conditions, right?	The teacher asks Julien what he got from the calculations he performed. He intervenes to provide an interpretation of the calculations. He points to the nature of the results as the "price of a pass relative to each of the three conditions".	The teacher gives an interpretation of each of the calculations performed by Julien. His didactical intention is to construct Julien's calculations as a support for introducing the stages of reasoning .	(1) Giving a statement the status of a "conclusion" in the development of a reasoning. Invitation to comment on Julien's results and to position them relative to an action. (2) Use of rhetorical didactical means:
4.5	Julien: Yeah!			Agreement, approval.
4.6	Teacher: (1) Okay, first conclusion after that?	The teacher questions Julien on what he gets from his calculations.	.	(1) Request to make an inference. The teacher waits for the student to continue his reasoning and articulate a conclusion.
4.7	Julien: And then I did…	No answer; Julien seems to want to continue to describe his calculations.		
4.8	Teacher: No, your first conclusion after that? When you were done with these calculation, what did you think to yourself?	The teacher reiterates his question.	The teacher makes a second attempt, with the same aim as in 4.4. But the formulation is more precise.	Recall of what is a conclusion; invitation to comment on a result.

4.9	Another student: Which one was less expensive.	A student puts into words the question that the teacher has previously asked in an implicit manner.	A student points to Julien what he could get from his calculations, namely a comparison of prices.	Question on an order relation. Project formulation.
4.10	Julien: Yeah! Which one was less expensive… But, no, I couldn't see…			But "which one" does not denote a well-determined object. A passive explanation. Impossibility to realize a project.
4.11	A student: But yes, you can see!	A student points out to Julien that he has all the necessary information.	The student pushes Julien to produce a reasoning, by pointing out to him that he has all the necessary elements to conclude (i.e. to compare the prices).	Possibility of realizing a project.
4.12	Julien: (1) Yes, it was 1275 (2) because a pass cost 5F (3) more or less and then (4) so then I tried, in short, I did 979 less 12, I got 967 and then I multiplied 967 by all the results of the divisions.	Julien gives the expected answer and continues to describe his calculations.	Julien articulates the conclusion, expected in the module 2. He goes back immediately to his initial reasoning, in describing his calculations.	(1) Implicit conclusion (2) Explanation (3) Estimation (4) Direct description of a sequence of actions and organization. Strategic or organizational reasoning, lexpressed orally.
4.13	Teacher: To find what?	The teacher questions Julien on the aim of his calculations.		Project; request to name a result. Request for an explanation.
4.14	Julien: To find the price of how much it was going to cost.	Julien points to the aim of his calculation: to calculate the total expense (for the students who were present at the trip).	Julien indicates the purpose or project he has in mind: for each group rate, to calculate the total expense.	Naming the result. Articulation of the purpose of his procedure.
4.15	Teacher: Yes, the price… to find which one was the least expensive.		The teacher starts from the formulation of the student and transforms it. Julien stated that his aim is to calculate the total expense for each of the three cases. But the teacher focuses on the comparison of the group rates. . The teacher will establish that Julien's	Rhetorical didactical means: Element of a local explicit reasoning of the teacher, which aimes at re-positioning the calculations in the perspective of the comparison of the three rates. Recall of the necessity to subordinate a result to the main task. Didactical intention: reject the calculations by making them appear as useless, redundant, with respect to the previously established conclusion.

			calculations are useless for the comparison of the rates.	
4.16	Julien: Yes.			Accord
4.17	Teacher: And you did the three calculations?	The teacher wishes to make Julien aware of the fact that the calculations were not necessary, that reasoning could help to avoid doing calculations.		Effectiveness of an action.
4.18	Julien: Yes.			
4.19	Teacher: It was necessary?			Call for a judgement of the relevance or adequacy of a calculation.
4.20	Julien: Well... yeah...			Agreement
4.21	Another student: To see which one was the least expensive.			Subordination recall, as in 4.15.
4.22	Teacher: You didn't know it before?	The teacher wants to incite Julien to reflect on his reasons for doing the calculations.	The intention would be: "could you know it beforehand, without doing the calculations?" It is, therefore, a call for a direct reasoning.	Call for the anticipation of the role of a result in the resolution of a problem. Call for a formulation of a direct local reasoning.
4.23	Julien: Yes, I knew it… but…	The student cannot distinguish between his opinion and the justification required by the teacher.		
4.24	Teacher: Okay then, so what is the result?	The teacher re-asks Julien to formulate his conclusion.		
4.25	Julien: So I saw which one was the least expensive, and then…			Validity status: subjective certitude

4.3 Discussion

The analysis of the implicit model of action allows us to identify the implicit mathematical model and Julien's representation of the objective situation. His model is that of the classical commercial situation, based on selling the passes per unit, corresponding to the mathematical model of proportionality.

The transcript (Table 1) shows that, in phase 7, Julien describes his calculations without providing the class with more explanations on why he did them. This is why his project is not accessible to the class, which makes it necessary for the teacher to intervene. By proceeding this way, he presents the teacher with the opportunity to interpret his calculations in a way which does not necessarily correspond to his (Julien's) initial project. The teacher grasps at this opportunity; using rhetorical didactical means, he manages to divert Julien's initial project to the benefit of his own, which is to develop the reasoning underlying Module 2 (comparison of the three rates) of the standard solution.

Moreover, our analysis shows that the teacher tries, several times, to engage a discussion on the validity of the presented procedures, or, more precisely, on the validity of the decisions underlying students' reasoning. However, his attempts all fail, one after another.

5. CONCLUSIONS AND CONJECTURES

5.1 Students' reasoning

The object of our analysis was the influence of certain features of the situation proposed to the students on the elaboration of the different forms of reasoning, their use and the possibilities of their processing available to the teacher during the whole class presentation and discussion of the solutions phase.

This analysis (see Table 1) shows that the forms of reasoning elaborated by the students were few, that they were not very complex in terms of the number of calculations and the number of stages involved.

This analysis implies that the teacher has no means for an effective processing of the produced reasoning, i.e. he cannot use logical reasoning directly related to the objective situation in arguing with the students' solutions.

This brings us to the first conjecture: the factor which constraints the teacher's possibilities of taking into account, articulating and processing students' reasoning is not so much the complexity of this reasoning but another feature which is related to the very nature of the situation proposed to the students.

5.2 The effect of the lesson on students' behavior and learning

5.2.1 The effect of the lesson on the validity of the reasoning and students' conviction

In the complete analysis of the transcript there is a lot of evidence that the students, having produced a reasoning based on a representation conforming to the teacher's expectations, have not become aware of the conditions which define the objective milieu. Indeed, in phase 7, they are unable to formulate the reasons that led them to elaborate these forms of reasoning, or even to react to the reasoning of their classmates when these are based on erroneous representations of the objective situation.

This can be partly explained by the fact that the situation does not provide the students with the possibility of testing their decisions: the objective milieu does not

respond with any feedback to the students' actions. Therefore the students have no means to validate or reject their reasoning and therefore to reflect on the decisions underlying their implicit models of action or their representations of the objective milieu.

5.2.2 The effect on the actions, language and opinions of the students

The students, unable to judge the validity of their work, cannot use the reasoning they have produced as arguments in a debate. The debate amongst peers wished for by the teacher is out of the students' reach.

5.3 The effect on the didactical process

5.3.1 The devolution

Decisions underlying the elaboration of each of the models are closely linked with the students' representations of the objective milieu. But this situation is not happening in real time and the students have to imagine the rules governing its functioning. Since the objective milieu is not clearly defined, this leads the students to construct different representations of the situation and therefore also different implicit models of action. Thus, the objective situation cannot be devolved to the students, i.e. the students cannot challenge the retail sales model adopted by the majority, or even calculate the results of the different possible choices.

5.3.2 Didactical corrections

The complete analysis of the transcript shows that the teacher cannot bring the students to articulate the reasons underlying their implicit models of action. To avoid a block, related to the fact that the students do not understand the decisions made by their peers, the teacher is forced to use rhetorical didactical means (Table 1). These means make it possible for the teacher to divert the initial project of a student to the benefit of his own, i.e. the establishment of certain modules of the standard solution. However, the real reasons that justify the elaboration of the module are not there for the students to see; the reasons which underlie and justify the connections between the data given in formulation of the problem situation are hidden.

6. FINAL CONCLUSIONS

The study shows that although the students, faced with a problem situation elaborated and conducted by the teacher, have certainly produced forms of reasoning, they have not made much progress in their practice of reasoning. Indeed, they have not reflected back on their reasoning, on its validity, relevance or adequacy because the teacher was not able to process it. He could not respond to this reasoning by logical arguments based on the objective situation; he was forced to use rhetorical means.

Now, it is not the complexity of the students' reasoning that forced the teacher to use this type of means but the fact that the problem situation could not be devolved to the students. This implies that it is not the teacher's management of the whole class presentation and discussion of the students' work that is challenged here, but rather

the nature itself of the situation set up by the teacher, which strongly constrains the possibilities of really taking into account the students' reasoning.

The objective situation does not make it possible for the teacher to bring the students to:

- share with their peers the real reasons that have led each of them to construct implicit models of action and take some decisions in the framework of the corresponding models;
- grasp the reasons why the steps of the expected, standard solution are necessary;
- share the reasoning underlying each module of the standard solution.

If a situation provides the teacher with the possibility of devolving to the students an "autonomous" (or "self-contained") situation of action, then, according to the theory of didactical situations in mathematics, during the phase of analysis of students' solutions the teacher can refer to the objective situation. This is because the students can develop their personal strategies and forms of reasoning related to the situations with which they are confronted. The teacher does not have to have recourse to rhetorical didactical means to process students' forms of reasoning.

If, on the other hand, the teacher has no such possibility, the teacher cannot refer in his arguments just to the objective situation and must bring in information and provide feedback on the basis of a project that is not visible for the students; and this is why he is forced to use rhetorical didactical means.

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