FROM ORAL TO WRITTEN TEXTS IN GRADE I AND THE LONG TERM APPROACH TO MATHEMATICAL ARGUMENTATION

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The aim of this paper is to elaborate on (and provide some long term experimental evidence for) the following hypothesis: during the first grade students' approach to writing texts, an appropriate educational setting (based on both social interaction managed by the teacher and students' involvement in well chosen concrete experiences) can give them the opportunity of developing important skills related to mathematical argumentation.

1. INTRODUCTION

In the last decade the early development of students' argumentative skills progressively became a subject of major concern for mathematics educators for different reasons: the need for an early approach to skills that are relevant in the proving process (under the pressure of curricular changes that brought to a re-evaluation of mathematical proof in school: see NCTM Standards, 2000); the exploration of the potential of social interaction in developing mathematical knowledge and skills (see Krummheuer, 1995; Yackel, 1998); the importance of argumentative skills in curricula aimed at enhancing students' intellectual autonomy (see Maher, 2002). On the other side, what we know about argumentative skills (starting from Piaget, 1923; 1947) implies that they cannot be developed within the narrow borders of one discipline (in particular, mathematics): students' argumentative potential needs to be nurtured across different activities, demanding a large amount of time. The contributions by E. Yackel, C. Maher and others mostly concern classroom argumentative activities about *mathematical* subjects in grades 3 or 4 and onwards. How to prepare the ground for these argumentative activities in earlier stages? This paper aims at elaborating (and supporting through experimental evidence) a working hypothesis concerning the early development of argumentative skills that are relevant for mathematical argumentation through a suitable management (see Subsection 4.1.) of the appropriation of written language by first grade students. Some educational implications for our hypothesis will be outlined in the Conclusion.

2. THEORETICAL FRAMEWORK

2.1. Argumentation and mathematical argumentation

I shall use the word "argumentation" both for the process that produces a logically connected (but not necessarily deductive) discourse about a subject, and its product (cf Boero, Douek & Ferrari, 2002, p. 250). *Mathematical argumentation* can be characterised as that peculiar kind of argumentation, which deals with mathematical

objects and skills (including general logical skills that are relevant in the mathematical discourse, like the management of the hypothetical reasoning). Hereafter, I consider some general attributes of argumentation (cf. Plantin, 1990) that are specially relevant in the case of *mathematical* argumentation.

i)- Production of a proposition that will be under discussion, in particular an interpretation, a guess, a plan, etc.; it may be produced to initiate an argumentation, or appear later as a partial result of the argumentation.

ii)- Production of reasons ("arguments") to validate the proposition - or question it. The reasons are taken from a reference corpus (in our experimental situations, the shared knowledge of the class - see Boero, Douek & Ferrari, 2002, p. 250, 256) they can be expressed using a number of representations (verbal statements, experimental evidences, drawings…). They can concern the use of peculiar tools (in the case of the discussion of a plan), the production of counter-examples (in the case of the discussion of a conjecture), etc.

iii)- Arguments and the proposition under scrutiny are held together by reasoning aimed at justifying, raising doubts, contradicting, refuting, interpreting, drawing new conclusions.

iv)- There is a global structure that needs to be maintained for the argumentation to be followed and understood. Verbal organisation is the visible aspect of such structure.

v)- The cognitive activity of a subject elaborating an argumentation is both conscious and voluntary; it presupposes the internalisation of an "other" who is in a position to control or regulate the logic of the reasoning, the truth of the statements, and the treatment of the signs involved.

2.2. **Oral and written texts**

There are two main (and rather coherent) references, one coming from Vygotsky's seminal work about the child's transition from oral communication to written text, the other related to Duval's investigation on the specificity of the written text in comparison with the oral text. Shortly, in Vygotsky's work (see Vygotsky, 1985) the transition from oral to written text is considered as a prototype for the transition from common knowledge to scientific knowledge in terms of consciousness, intentionality and systematic organisation. This is related to the fact that a written text must address a distant "other" in quite different conditions from oral communication, where the partners can understand each others with hints, gestures and various means of non verbal communication. Duval in his contribution (Duval, 1999) points out some characteristics of cognitive processes underlying writing (compared to oral communication): writing needs a greater conscious control than speaking; it needs a reorganisation of the oral text; it allows to escape the constraints of the oral text (as concerns short term memory, evidences, etc.)

Both Vygotsky's and Duval's contributions support and legitimate the hypothesis that the transition from oral to written texts could provide an opportunity to develop some argumentative skills relevant to mathematical argumentation (cf 2.1: iv, v).

2.3. Social interaction in the classroom

In this paper I will consider social interaction as it works in communication situations designed with the purpose of developing linguistic representation of knowledge. This aspect has been widely considered in current literature over the last two decades (cf. Steinbring et al., 1998 for a representative set of orientations in the field of mathematics education). I will consider communication as a condition for cultural development for the individual (see Episode 2 in Section 3) and for the group of which one is a part (see Episode 3). In a Vygotskian perspective, communication reflects and influences the development of thought (see Vygotsky's comments about Piaget's internal language: Vygotsky, 1985, Ch. 2). In particular, argumentation in communication situations can enhance the development of students' argumentative skills considered in Subsection 2.1 (specially, see v).

2.4. Context

Wedege (1999) discusses the use of the word "context" in educational literature and proposes a distinction between "situation context" (e.g. workplace, classroom social context, computer learning environments, etc.) and "task context" (e.g. everyday life situations evoked in a problem-solving task). We can remark that some studies concerning the opportunities offered by "task contexts" are coherently conceived from a Vygotskian perspective of "social construction of knowledge", i.e. take also the "situation context" into account as a relevant issue (for an example, see Bartolini Bussi et al, 1999). This will be also the case of the study reported in this paper. Concerning the potential of the task context in terms of the development of argumentative skills related to logical reasoning (cf ii and iii, Subsection 2.1), I will refer to Guala & Boero (1999) and Arzarello (2000). They have pointed out the potential of time and space constraints inherent in the task context for the development of logical skills

3.THE MAIN HYPOTHESIS

The preceding theoretical considerations legitimate the following hypothesis: an interactive management of students' approach to writing can offer students the opportunity of approaching argumentative skills, relevant for mathematical argumentation, provided that the following conditions are fulfilled:

- 1-1 teacher-student interactions and classroom discussions orchestrated by the teacher are aimed at transforming students' utterances into pieces of written texts through explicit prompts by the teacher (and/or more competent peers) motivating the changes to be made;
- suitable tasks are chosen, based on concrete operations related to familiar task contexts that ensure the possibility of an immediate feedback for students' mistakes and incomplete texts, and are rich in logical connections related to space and time constraints.

4. EXPERIMENTAL EVIDENCE

4.1. Source of data

The Genoa Group for research in mathematics education has developed an innovative methodology for the approach to writing in first grade classes, within their project for an integrated teaching of mathematics and other disciplines in primary school (see Boero, 1994; Boero et al., 1995). This Project is conceived in the perspective of "research for innovation" (see Arzarello & Bartolini Bussi, 1998). Teachers interact individually with students about their experiences (mainly everyday life experiences - like the use of machines, or easy productions of objects and food - which ensure a concrete feedback for what children say). The student's utterances are interpreted by the teacher, who gradually helps the student to improve them in order to get a text suitable for writing. At the end of this process, the student dictates the oral text to the teacher; finally, the student copies the text, written by the teacher, on his copybook. Frequently, some of the texts produced during the 1-1 teacher-student interactions become an object of discussion for the whole class, in order to further improve them. Sometimes the production of a text related to a common experience (mainly in the "technological" domain) is proposed as a collective task for the whole class, through a discussion orchestrated by the teacher (who tries to get interactively a written text starting from the students' utterances: see Episode 3). This gradual introduction to written texts with an essential role given to the teacher's mediation and its rooting in student's shared concrete experience is an educational setting inspired by Vygotsky's elaboration on the dialectics between "ordinary" knowledge and "scientific" knowledge (cf Subsections 2.2, 2.3 and 2.4; see Boero et al., 1995).

The following four episodes come from first grade classes that adopt the Genoa Group Project. The first episode fulfils only a general introductory function. The fourth episode shows how skills developed in the previous years intervene in a situation rich in geometrical content.

4.2. Some episodes

Episode 1: End of January, grade I. Interaction between the teacher (T) and Maria (a low achieving student who is approaching the production of written texts). Students have already learned to report orally on some easy, concrete procedures.

(T): Tell me, Maria, how you have produced the soap bubbles

(Maria): I put the soap solution in the glass, then I blew into the glass, and the bubbles came out

(T): But if you blow into the glass, no bubble comes out: do it!

(Maria): I have forgotten to say that I blow into the soap solution through a straw

(T): OK; now you can tell me with precision how you have produced the soap bubbles

(Maria): I put some soap solution in the glass, then I took the straw and I blew through it, and the bubbles came out

(T): (repeats Maria's phrases) OK; now you can dictate your text to me. Remember that you must speak very slowly, in order to give me enough time to write.
(Maria dictates a text that is very near to her oral text, and the teacher writes it down)
(T): (slowly reads the whole text, then concentrates on a particular sentence): Maria, pay attention: "I took the straw and I blew through it": If I take the straw and I blow through it, (the teacher performs the action) no bubble comes out!
(Maria): I have not said that the straw was put into the glass...
(T): Into the glass?
(Maria): No, into the soap solution!
(T): Did you put the whole straw into the soap solution?
(Maria): No, only the end!
(T): OK, now you can dictate the right sentence (etc.)

Comments about this episode: Let us consider the whole dialogue as a discourse. The teacher kept the line of the discourse (cf iv, Subs. 2.1): all the data (elements of the experience) that played a role had to be made explicit, and in the right order; logical reasons (cf ii, iii, Subs. 2.1) were given by the teacher to justify his requests (some of them may be seen as verbal logical reasons, some as related to the "logic" of the events). No argumentation is produced by the student, although the activity is not only an introduction to the production of written texts, but also to some rules for producing texts and related skills that are relevant for mathematical argumentation.

Episode 2: End of March, grade I. Students have to plan how to assemble a toy windmill; they can see a toy windmill already assembled, and the pieces to be assembled to get another toy windmill. Stefania is an average-achieving student; she is already able to write texts; the 1-1 interaction with the teacher concerns a text written by her.

(Stefania wrote: "I will put the nail into the wood stick, then I will put the propeller and the washers").
T: Let us try to do it. I take the nail and I put its tip into the wood stick...
(she mimics the action)
(Stefania): It does not work... How can I put the propeller?
(T): Explain why your text does not work.
(Stefania): Because if I put the nail into the wood stick, then I cannot put the propeller, because the head of the nail does not allow the propeller to go over the nail. And also the washers cannot go over. I must put one washer, then the propeller, then the other washer... Then I can put the nail into the wood stick.
(T): OK, write a new text, including the explanation of the reason why you need to postpone putting the nail into the wood stick.
(Stefania's new text: "I will put one washer over the nail, then the propeller, then the other washer. Then I will put the nail into the wood stick. If I had put the nail into the wood stick at the beginning, it would have been impossible to put the washers and the propeller over the nail".

Comments about this episode: the episode shows how in a first grade class the teacher can provoke awareness about logical requirements concerning a space-time situation. The teacher suggests an argumentative style to the students, and the student internalises it when she is writing the second version of her text. Here the student produces an argumentation that involves relevant skills in a mathematics education perspective (cf ii, iii, iv Subs. 2.1). Reasoning (even at the metacognitive level) is consciously carried out by her, under the pressure of the teacher who leaves to her part of the responsibility for leading the argumentation : Stefania has to explain why her text does not work. The teacher helps her to find the reason why it does not work, as if she suggested a counter example through material facts. The student puts it in words and draws conclusions about the structure the text should take; she becomes more conscious of the logical role of the elements of the experience within the text, as in the case of the construction of an argumentation aimed at validating the solution of an applied mathematical problem.

Episode 3: End of March, grade I. A classroom discussion regarding how to prepare a report for an absent schoolmate about some activities that concerned temperatures.

(T): during the last week Fatima was ill; now she has recovered, and in two or three days she will be back with us. Last week we have made a lot of work with the thermometer. It would be good to prepare a text to explain to her what we have done, and why. Remember: we must explain why we have decided to use the thermometer, and how. Her mother will help her understand the text, but her mother was not here, so it is necessary to be extremely clear in preparing our text [(I)]. What should we write in this text?

(as usual in the construction of a synthesis about classroom work, the teacher goes to the big blackboard and writes down what students propose)

(Debora): Dear Fatima, we have learned to use the thermometer

(Ugo): But Fatima does not know why we have decided to use the thermometer

(Bianca): We could write: we have decided to use the thermometer to get a true... to be able to say if it is true...[(II)]

(Ugo) That today it is warmer than yesterday

(Daniele) Because there was somebody who told that it was warmer, and somebody who told that it was colder [(III)]

(Luca) And somebody told that it was warm, while I told that it was rather cold

(T): Now we can try to write the first sentence, about why we have decided to use the thermometer

(Debora): We have decided to use the thermometer in order to establish if it is really cold or warm, out of our impressions

(Ugo) And if today it is colder or warmer than yesterday

(the teacher writes the whole sentence, coming from Debora's and Ugo's contributions, on the blackboard; then she reads it very slowly)

(T): but... we must compare... we must only establish that today it is warmer or colder than yesterday... And next monday?

(Daniele) Today ... not only today, anyone day [IV]
(Patrizia) Otherwise Fatima could imagine that we have made a lot of work only for one single day. [V]
(Matteo) It is.. it is for all days: Monday, Tuesday, ...
(T): How to modify our sentence? (she reads the sentence slowly)
(Ivan: he reads the text on the blackboard): We have decided to use the thermometer in order to establish if it is really cold or warm, out of our impressions, and if ...if one day is colder or warmer than yesterday...
(Daniele): Not, not yesterday... the day... the day that comes before...
(Ugo): like yesterday for before today, and the day before yesterday for yesterday
(Daniele): The preceding day [VI]
(Ugo): Yes, the preceding day
(T): Daniele, please, say the whole sentence
[...]

Comments about this episode: This fragment shows how the task of producing a classroom report (in a co-operative style, with an emotionally shared communication purpose) about the "why" and "how" of a shared activity can provide students with the opportunity of an intensive argumentative activity, during the elaboration of the text and, in particular, the production of a sentence that carries generality. The final text is rich in argumentative skills (specially related to ii: see Subs. 2.1) that are relevant for mathematics education, in particular in the case of mathematical argumentation dealing with mathematical modelling and applied mathematical problem solving. In particular we may observe (cf. [II] and [III]) how the meaning of a tool and its contextualisation come into play. This aspect is very important in a mathematical argumentation when arguments for the adequacy of a tool to an activity (or a plan, or a solution) must be produced. Also skills related to the management of the generality of mathematical statements are involved. At the points [IV], [V], [VI] we can observe a progressive focusing on a problem of generality, up to its condensed, appropriate expression. Concerning v) (Subs. 2.1), we may observe at the point [I] how the didactical contract takes on the role of the idealised internalised "other" that controls the production of an argumentation.

Fourth episode (third grade class):

In this situation an argumentation, rich in mathematical content, is produced within a non-mathematical context. The teacher organises a classroom discussion about how to set a wood table on the ground, in order to place it horizontally. Discussion starts by considering a student's text (copied by the teacher on the blackboard):

"I put the spirit-level on the table, then I look at the bubble: if it is at the centre of the glass, then it means that the table is horizontal (Daniele)"

(T): We must help Daniele to improve his text by explaining to him his mistakes and lacks, and suggesting appropriate changes. The final text will be used to explain our work to another class that has not enough time to make all the activities we perform.
(Rosa): Daniele, it is not sufficient to check if the spirit-level is horizontal... It can be horizontal, and the table is not horizontal. VII)
(Daniele): But we have seen that if we put the spirit level on the table, and the spirit-level is horizontal, then the table is horizontal.
(Federico): Let us try again... (he takes the table and puts it on the floor). Here is the table on the floor: the table is horizontal and the spirit-level... Look at it, the bubble is perfectly in the mean middle of the glass... The spirit-level is horizontal! VIII)
(Rosa): But I have said that if the spirit-level is horizontal, the table can be inclined. It is clear that if the table is horizontal, also the spirit-level is horizontal!
(Stefano): It seems to me that... I do not understand the difference between Rosa and Federico: Federico says that the table is horizontal and the spirit-level is horizontal... Also Rosa says that if the table is horizontal, the spirit-level is horizontal. IX)
(Rosa): But I have said that the contrary... the contrary is not true, if the spirit level is horizontal... the table can be inclined. Look at this: (she slowly tilts the table, so that the bubble remains at the centre of the glass). X)
Do you see? The bubble is still in the centre... the spirit-level is horizontal, but the table is not horizontal!
(Miriam): Rosa is right! The spirit-level is horizontal, but the table is not horizontal!
(Daniele): But if I move the spirit-level on the table, it is no more horizontal!
(T): If Daniele moves the spirit-level on the table, it is no more horizontal!
(Robi): Now I remember! I had put the spirit-level like this and like this (he indicates two different directions with his hands).
(Daniele): Yes, I meant that the spirit-level must be horizontal... must be horizontal in any direction!
(T): Daniele, read your text and try to modify it in order to get a more precise text.
(Daniele): "I put the spirit-level on the table, then I look at the bubble: if it is at the centre of the glass, then it means that the table is horizontal" . I must say: "I put the spirit-level on the table in different positions, each time I look at the bubble: if it is always at the centre of the glass, the table is horizontal"
(the teacher writes the second text dictated by Daniele below his first text)
(T): Two remarks. First, you have used the word "positions" (she underlines "positions"); when you spoke, you had used the word "direction" (she writes "direction" under the word "positions". Is it the same? Second, it would be good to explain why it is so important to use the spirit-level in different positions – (or directions?). (etc)

Comments about this episode: here we say how the preceding systematic practice of argumentation related to the production of written texts in a social interaction situation allows third grade students to engage almost autonomously in an important discussion involving relevant geometric (and logical) content. Now the teacher pulls students to leave a trace of the collective argumentation in the written text: not only the production of the text is an opportunity to develop argumentation, but the production of a rather complex argumentative written text becomes an explicit target for classroom discussion. For further details about this kind of situations in the same task context, see Douek, 1999. See also Bartolini Bussi et al., 1999 for other examples in different task contexts. As concerns specific behaviours that are relevant for mathematical argumentation, we may remark that at the point VII) Rosa succeeded in spotting and expressing an important flaw in Daniele's text. Federico (see VIII)) was not able to grasp the delicate logical relation and its negation, but Rosa and Stefano extracted the problem of negation from the whole discussion and clarified it (see IX)) We recognize here important moves that are relevant in mathematical argumentation: finding logical flaws, expressing and clarifying them by focusing on them (cf. iv, Subs. 2.1). Concerning the point X) we may remark how giving examples and counter-examples contributes to clarify the problem under scrutiny and provides important steps in an argumentation (cf ii) and iii), Subs. 2.1). Here the steps of reasoning are logical and non-trivial, and the arguments are mathematical even though the language is not specialised. Finally, at the end of the excerpt we may remark that the teacher intervenes on the student's text in order to get to a new statement that corresponds to observed facts. A strong effort of interpretation (relevant by itself, as well as in the perspective of taking part in an argumentative activity) is demanded.

4. CONCLUSION

Both theoretical reasons (in particular, Vygotsky's and Duval's contributions about writing in comparison with speaking) and experimental evidence seem to support the working hypothesis stated in this paper. Indeed an early, intensive approach to argumentative skills, relevant for mathematical argumentation, seems to be possible through an interactive management of students' approach to writing and classroom discussions about produced texts (provided that suitable tasks are chosen, based on concrete operations that ensure the possibility of an immediate feedback for students' flaws). The episodes show in particular how space and time constraints inherent in the task context intervene in the development of students' argumentative skills, either as sources of mistakes or incomplete statements to be detected and overcome (see the second and the fourth episode), or as opportunities to deal with generality and express it (see the third episode).

The hypothesis dealt with in this paper opens an interesting perspective for the intervention on students' argumentative skills: indeed students' access to writing texts is the crucial goal for teachers in first grade. A synergy between the achievement of this goal and the development of students' argumentative skills,

relevant for mathematics education, is one possible outcome of the study reported in this paper.

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